Diff Cal Practice Problems for Test #3

1. Locate the absolute extrema of the function on the closed interval. $f(x) = x^3 - \frac{3}{2}x^2$, $[-1, 2]$

2. Determine if Rolle's Theorem can be applied to f on the closed interval [a , b]. If Rolle's Theorem can be applied, find all values of c in the open interval (a, b) such that f'(c)=0.

$$f(x) = (x-3)(x+1)^2 \ , \qquad [-1, 3]$$

3. Determine whether the Mean Value Theorem can be applied to f on the closed interval [a, b]. If the Mean Value Theorem can be applied, find all values of c in the open interval (a, b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$. $f(x) = x(x^2 - x - 2)$, $[-1, 1]$

4. Find the open intervals on which the function is increasing or decreasing and locate all relative extrema. $f(x) = (x+2)^2(x-1)$

5. Find the open intervals on which the function is increasing or decreasing and locate all relative extrema. $y = \frac{x^2}{x^2-9}$

6. Find the points of inflection and discuss concavity of the graph of the function. $f(x) = x^3(x-4)$

7. Find the points of inflection and discuss concavity of the graph of the function. $f(x) = \frac{x}{x^2+1}$

8. Use the Second Derivative Test to find all relative extrema. $f(x) = x^2 - 6x + 7$

9. Find the limit. $\lim_{n\to\infty} \frac{2x^3-5x+1}{4x^3-3x^2+x+25}$

10. Find the limit. $\lim_{n\to-\infty} \frac{-2x+5}{\sqrt{x^2+2x}}$

11. Find the horizontal asymptotes. $f(x) = \frac{5x}{\sqrt{x^2+5}}$

12. The radius of a right circular cylinder is given by $\sqrt{t+2}$ and its height is $\frac{1}{2}t$, where t is time in seconds and the dimensions are in inches. Find the rate of change of the volume with respect to time. Volume of a cylinder is given by $V = \pi r^2 h$, where r is the radius of the cylinder and h is the height.

13. A conical tank is 10 feet across at the top and 10 feet deep. If it is being filled with water at a rate of 5 cubic feet per minute, find the rate of change of the depth of the water when it is 3 feet deep. The volume of a cone is given by $= \frac{1}{3}\pi r^2 h$, where r is the radius of the cone and h is the height. Give an exact answer in terms of π.

14. The radius of a sphere is expanding at a rate of 3 centimeters per second. Find the rate of change of the volume of the cube when the radius is 12 centimeters.

15. Find the limit (using l'Hôpital's rule). $\lim_{x\to 0} \frac{e^x-(1-x)}{x}$

16. Find the limit (using l'Hôpital's rule). $\lim_{x\to\infty} \frac{x^2}{\sqrt{x^2+1}}$

17. Find the limit (using l'Hôpital's rule). $\lim_{x\to\infty} \frac{\sin x}{x-\pi}$

18. Find the limit (using l'Hôpital's rule). $\lim_{x\to\infty} \frac{\ln x^4}{x^3}$