$\mathbb{N}$ = natural #'s = counting #'s

Distributive Property: $a(b+c)=ab+ac$

additive inverse of x is $-x$

$\forall$ = forall ; $\exists$ = there exists

$\exists!$ = there exist unique

$\in$ = is an element of

$\subset$: is a subset of

0 is the additive identity: $0+x=x$

1 is the multiplicative identity

$(a-b)^2=a^2-2ab+b^2$

If $a=b$ & $b=c$, then $a=c$
(transitive property of equality)

$(a-b)(a+b)=a^2-b^2$

multiplicative inverse of x is $\frac{1}{x}$

commutativity: $ab=ba$

associativity: $(ab)c=a(bc)$

Using Factoring by Grouping to Factor Quadratic Trinomials

$2x^2 \underline{-11x} - 40$

What factors of $2(-40)=-80$ sum to give us -11?

5 & -16

$2x^2+\underline{5x-16x}-40$

$x(2x+5)-8(2x+5)$

$\boxed{(2x+5)(x-8)}$

Factors of -80:

~~1·80~~

~~2·40~~

~~4·20~~

5·16

$$4x^2 + 9x + 10$$

what factors of $4(10) = 40$ sum to $+9$?

there aren't any!

$\Rightarrow$ this trinomial is not factorable over the integers!

$1 \cdot 40$
$2 \cdot 20$
$4 \cdot 10$
$5 \cdot 8$

$$10x^2 - 29x + 10$$

what factors of $10(10) = 100$ sum to -29? -25 & -4

$$\underbrace{10x^2 - 25x}\ \underbrace{-4x + 10}$$

$$5x(2x-5) - 2(2x-5)$$

$$\boxed{(2x-5)(5x-2)}$$

$$\frac{2x}{4y^2} + \frac{y}{3x} = \frac{2x}{4y^2} \cdot \frac{3x}{3x} + \frac{y}{3x} \cdot \frac{4y^2}{4y^2} =$$

$$= \frac{6x^2}{12xy^2} + \frac{4y^3}{12xy^2} = \frac{6x^2 + 4y^3}{12xy^2}$$

$$= \frac{\cancel{2}(3x^2 + 2y^3)}{\cancel{2}(6xy^2)} = \boxed{\frac{3x^2 + 2y^3}{6xy^2}}$$

$$\frac{2xy}{3} - \frac{4}{9x^2y}$$

$$= \frac{2xy}{3} \cdot \frac{3x^2y}{3x^2y} - \frac{4}{9x^2y}$$

$$= \frac{6x^3y^2}{9x^2y} - \frac{4}{9x^2y} = \boxed{\frac{6x^3y^2 - 4}{9x^2y}}$$

$$\frac{2}{x} + \frac{3}{x+1} = \frac{2}{x} \cdot \frac{(x+1)}{(x+1)} + \frac{3}{(x+1)} \cdot \frac{x}{x}$$

$$= \frac{2x+2}{x(x+1)} + \frac{3x}{x(x+1)}$$

$$= \frac{2x+2+3x}{x(x+1)} = \frac{5x+2}{x(x+1)} = \boxed{\frac{5x+2}{x^2+x}}$$

$$\frac{2}{x} \cdot \frac{3}{(x+1)} = \frac{2 \cdot 3}{x(x+1)} = \boxed{\frac{6}{x^2+x}}$$

$$\frac{x+3}{x^2-4} + \frac{2}{x-2} = \frac{x+3}{(x-2)(x+2)} + \frac{2}{(x-2)} \cdot \frac{(x+2)}{(x+2)}$$

$$= \frac{x+3+2x+4}{(x-2)(x+2)} = \boxed{\frac{3x+7}{x^2-4}}$$

$$\frac{10}{12} = \frac{\cancel{2} \cdot 5}{\cancel{2} \cdot 6}$$

$$\frac{10x^2y^3}{12x^3y} = \frac{\cancel{2} \cdot 5 \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{y} \cdot y \cdot y}{\cancel{2} \cdot 6 \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot \cancel{y}} = \boxed{\frac{5y^2}{6x}}$$

Rules of Exponents

$$X^n \cdot X^m = X^{n+m}$$

$$\frac{X^n}{X^m} = \frac{X^{n-m}}{1} = \frac{1}{X^{m-n}}$$

$$\frac{X^3}{X^4} = \frac{X^{3-4}}{1} = \frac{X^{-1}}{1} = \frac{1}{X^{4-3}} = \frac{1}{X}$$

$$X^{-n} = \frac{1}{X^n} \quad ; \quad \frac{1}{X^{-n}} = X^n$$

$$X^{-1} = \frac{1}{X^1} \quad ; \quad X^{-5} = \frac{1}{X^5}$$

$$\frac{6x^3y^5z^1}{8x^1y^4z^3} = \frac{3x^{3-1}y^{5-4}}{4z^{3-1}} = \boxed{\frac{3x^2y}{4z^2}}$$

$$a^2-b^2=(a-b)(a+b)$$
$$x^2-3^2=(x-3)(x+3)$$

$$\frac{2x^2-1}{x+3}+\frac{x}{x^2-9}$$

$$\frac{(2x^2-1)}{(x+3)}\cdot\frac{(x-3)}{(x-3)}+\frac{x}{(x+3)(x-3)}$$

$$\frac{2x^3-6x^2-x+3}{(x+3)(x-3)}+\frac{x}{(x+3)(x-3)}$$

$$\frac{2x^3-6x^2\cancel{-x}+3\cancel{+x}}{(x+3)(x-3)}=\boxed{\frac{2x^3-6x^2+3}{(x+3)(x-3)}}=\boxed{\frac{2x^3-6x^2+3}{x^2-9}}$$

Domain: $\mathbb{R}$ except ± 3