

HW #1

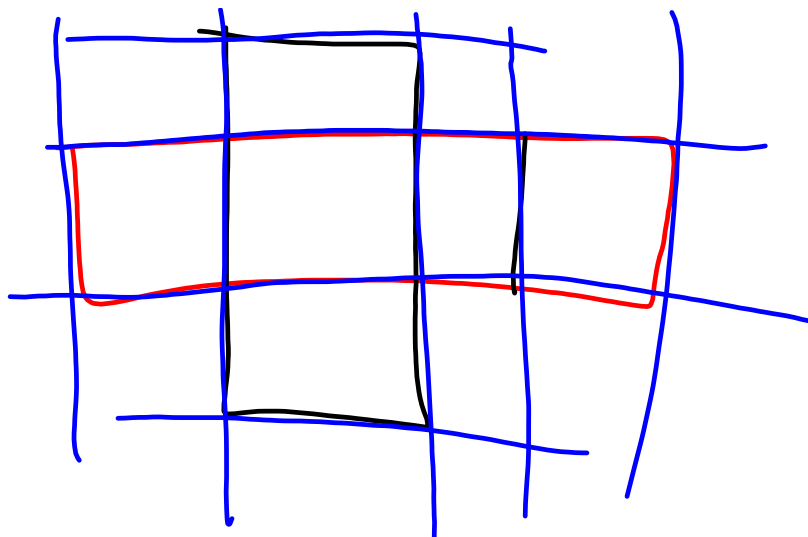
- Read Ch 1
- **Ch 1 Review Problems pp. 36-38 #1-30 - due FRIDAY 11/6**
- Start working on Geometry badge on Khan Academy; make sure you've added me as a coach using code listed on brewermath.com!

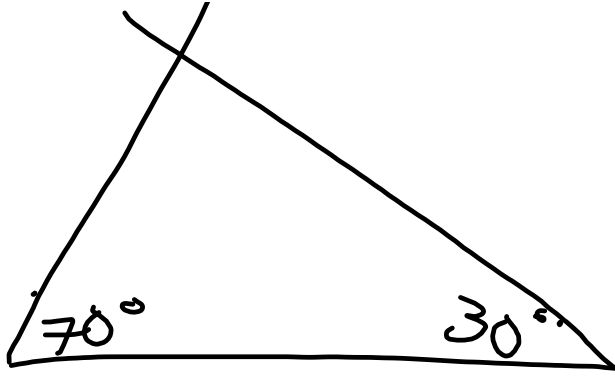
Quiz #1

- Vocab
- Fill in the blank proofs

HW #2

- Read Ch 2 & Ch 3
- **Ch 2 Review Problems pp. 71-74**
- **Ch 3 Review Problems pp. 124-128**
- Khan Academy exercises:
 - > "Introduction to Euclidean Geometry"
 - > "Angles and intersecting lines"





21. At a sports banquet there are 100 famous athletes. Each one is either a football player or a basketball player. At least one is a football player. Given any two of the athletes, at least one is a basketball player. **How many of the athletes are football players, and how many are basketball players?** Construct an indirect argument to explain your reasoning.

Theorem: There are 99 basketball players and only one football player.

Proof:

1. Suppose there are 2 or more football players.
2. If there are 2 or more football players then there exists a pair of players such that neither is a basketball player.
3. This contradicts that given any 2 players, at least one is a basketball player. Hence our assumption is false and therefore there is only one football player.

2.5 – A Deductive System

To avoid circular definitions, mathematics leaves certain terms undefined.

Those which we have seen so far include: point, line, plane.

These undefined terms can be used to define other terms, for example,

Def: Points are collinear iff there is a line that contains all of them.

Def: Lines are concurrent iff they contain the same point.

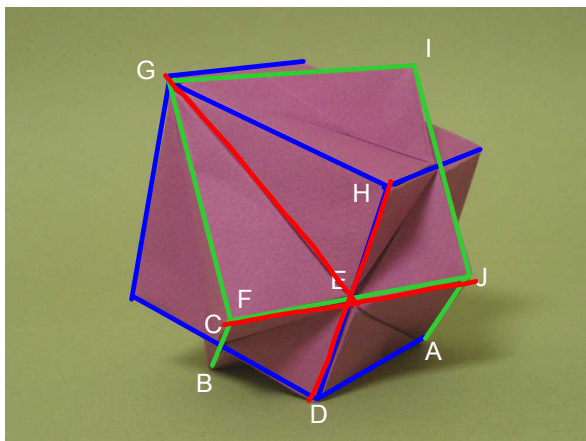
Just as it is impossible to define everything without going around in circles, it is impossible to prove everything. We leave some statements unproved, and use them as a basis for building proofs of other statements.

Def: A postulate is a statement that is assumed to be true without proof.

axiom

Postulate 1: Two points determine a line.

Postulate 2: Three noncollinear points determine a plane.



Determine if the following statements are true or false:

19. Points B, C, and F are collinear. **T** \overline{BF}

20. Points B and C determine a line. **T**

21. Points F, E, and J are coplanar. **T**

22. Points F, E, and J determine a plane. **F (collinear)**

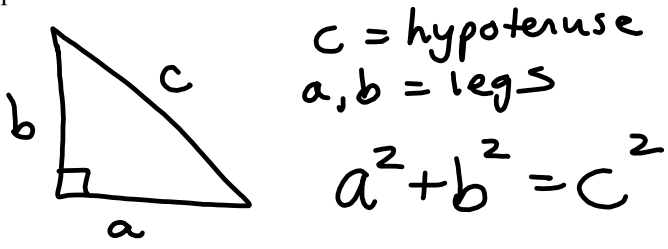
23. Points A, E, and G are collinear. **F (but they are coplanar)**

24. Points A, B, C, and J are coplanar. **T**

25. Lines DH, FJ, and EG are concurrent. **T (@ pt E)**

2.6 - Some Famous Theorems of Geometry

The Pythagorean Theorem: The square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides.



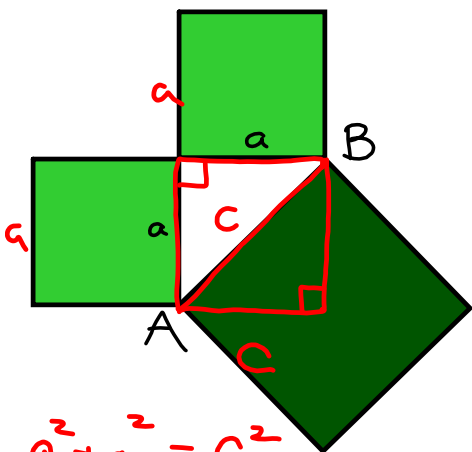
The Triangle Sum Theorem: The sum of the angles in a triangle is 180° .

Circle Theorems:

If the diameter of a circle is d , then its circumference is πd .

If the radius of a circle is r , then its area is πr^2 .

$d = 2r$
 $C = 2\pi r = \pi d$
 $A = \pi r^2$



$a^2 + a^2 = c^2$
 $2a^2 = c^2$

34. The area of a light green square is

a^2

35. The combined area of the two light green squares is

$a^2 + a^2 = 2a^2$

36. The area of the dark green square is

$c^2 = 2 \cdot a^2$
 $= \text{twice area of original square}$

Theorem:

41. "The area of a circle is half of the circumference multiplied by half of the diameter."
 ~ 6th century Indian astronomer Aryabhata

Is this true? Thm: $\pi r^2 = \frac{1}{2}C \cdot \frac{1}{2}d$

Proof:

1. $d = 2r$
2. $C = \pi d = 2\pi r$
3. $\frac{1}{2}C \cdot \frac{1}{2}d = \frac{1}{2}(2\pi r) \cdot \frac{1}{2}(2r) = \pi r \cdot r = \pi r^2$

3.1 – Number Operations and Equality

Algebraic Postulates of Equality:

Reflexive Property: $a = a$ (Any number is equal to itself.)

Substitution Property: If $a = b$, then a can be substituted for b in any expression.

Addition Property: If $a = b$, then $a + c = b + c$

Subtraction Property: If $a = b$, then $a - c = b - c$.

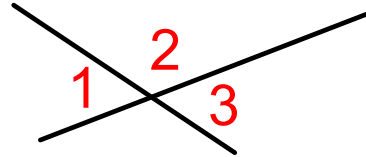
Multiplication Property: If $a = b$, then $ac = bc$. , $c \neq 0$

Division Property: If $a = b$, then $a/c = b/c$. , $c \neq 0$

State the property of equality illustrated by each statement:

3. If $c/d = \pi$, then $c = \pi d$ multiplication (by d)
4. If $\angle A + \angle B + \angle C = 180^\circ$ and $\angle C = \angle A + \angle B$, then $\angle C + \angle C = 180^\circ$. Substitution
5. If $2\angle C = 180^\circ$, then $\angle C = 90^\circ$. division (by 2)

This figure shows two lines intersecting to form several angles, three of which are numbered.



8. If $\angle 1 + \angle 2 = \angle 2 + \angle 3$, then $\angle 1 = \angle 3$. Why?

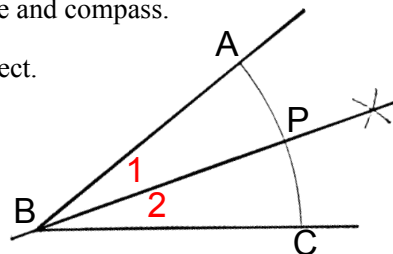
Subtraction (< 2)

9. If $\angle 1 = \angle 2$ and $\angle 2 = \angle 3$, then $\angle 1 = \angle 3$. Why?

Substitution

This figure shows how we bisected an angle by using a straightedge and compass.

Let's check the algebra to see that $\angle 1$ is the size that we would expect.



11. If $\angle ABC = \angle 1 + \angle 2$ and $\angle 1 = \angle 2$, then $\angle ABC = \angle 1 + \angle 1 = 2\angle 1$. Why?

Substitution (& simplification)

12. If $\angle ABC = 2\angle 1$, then $\angle 1 = (1/2) \angle ABC$. Why?

Division (by 2)

Quadratic formula

If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

37. What is the hypothesis of this theorem?

$ax^2 + bx + c = 0$

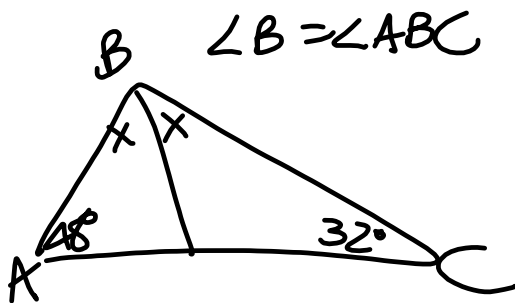
Name the postulate that is the reason for each of the following first three steps in its proof:

38. If $ax^2 + bx + c = 0$, then $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$. *division (by a)*

39. If $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$, then $x^2 + \frac{b}{a}x = -\frac{c}{a}$. *subtraction (by $\frac{c}{a}$)*

40. If $x^2 + \frac{b}{a}x = -\frac{c}{a}$, then $x^2 + \frac{b}{a}x + (\frac{b}{2a})^2 = -\frac{c}{a} + (\frac{b}{2a})^2$. *addition (by $(\frac{b}{2a})^2$)*

41. What kind of proof begins like this? *Direct proof (syllogism)*



$$\begin{aligned} \angle A + \angle B + \angle C &= 180^\circ \\ 48^\circ + \angle B + 32^\circ &= 180^\circ \\ \angle B &= 180^\circ - 48^\circ - 32^\circ \end{aligned}$$

$$\angle B =$$

$$\begin{aligned} x + x &= \angle B \\ x &= \frac{1}{2} \angle B \end{aligned}$$