

Theorem 17: Equal corresponding angles mean that lines are parallel.

Corollary 1: Equal alternate interior angles mean that lines are parallel.

Corollary 2: Supplementary interior angles on the same side of a transversal mean that lines are parallel.

Corollary 3: In a plane, two lines perpendicular to a third line are parallel.

The Parallel Postulate – Through a point not on a line, there is exactly one line parallel to the given line.

Theorem 18: In a plane, two lines parallel to a third line are parallel to each other.

Theorem 19: Parallel lines form equal corresponding angles.

Corollary 1: Parallel lines form equal alternate interior angles.

Corollary 2: Parallel lines form supplementary interior angles on the same side of a transversal.

Corollary 3: In a plane, a line perpendicular to one of two parallel lines is also perpendicular to the other.

Theorem 20: The Triangle Sum Theorem – The sum of the angles of a triangle is 180° .

Corollary 1: If two angles of one triangle are equal to two angles of another triangle, the third angles are equal.

Corollary 2: The acute angles of a right triangle are complementary.

Corollary 3: Each angle of an equilateral triangle is 60° .

Theorem 21: An exterior angle of a triangle is equal to the sum of the remote interior angles.

Theorem 22: The AAS Theorem – If two angles and the side opposite one of them in one triangle are equal to the corresponding parts of another triangle, the triangles are congruent.

Theorem 23: The HL Theorem – If the hypotenuse and a leg of one right triangle are equal to the corresponding parts of another right triangle, the triangles are congruent.

SSS, SAS, AAS, ASA, HL

7.1 – Quadrilaterals

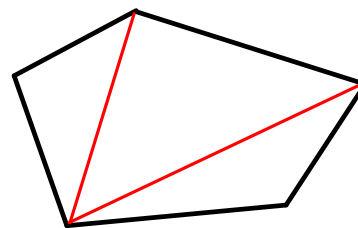
Def: A diagonal of a polygon is a line segment that connects any two nonconsecutive vertices.

Theorem 24: The sum of the angles of a quadrilateral is 360° .

Def: A rectangle is a quadrilateral each of whose angles is a right angle.

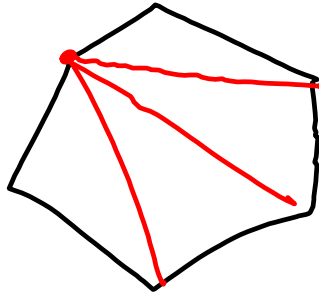
Corollary to Theorem 24: A quadrilateral is equiangular iff it is a rectangle.

In the figure below, a pentagon has been divided into triangles by the diagonals from one vertex.



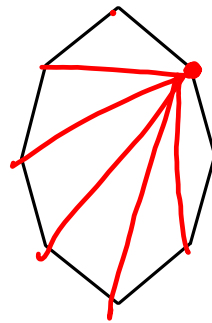
38. How many sides does a (5-gon) have,
 how many diagonals were drawn,
 and how many triangles were formed?
5
2
3

39. Draw a hexagon and the diagonals from one vertex.



40. How many sides does a hexagon have, **6**
 how many diagonals did you draw, **3**
 and how many triangles were formed? **4**

41. Draw the diagonals from one vertex for the given figure.



(Octagon)
 42. How many sides does the polygon have, **8**
 how many diagonals did you draw, **5**
 and how many triangles were formed? **6**

In general, if a polygon has n sides, in terms of n ,

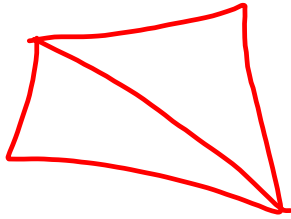
43. how many diagonals can be drawn from one vertex?

$$n-3$$

44. how many triangles do these diagonals form?

$$n-3+1 = n-2$$

45. Show that your answers are correct for a quadrilateral.



$$\begin{aligned} n &= 4 \\ n-3 &= 1 \checkmark \\ n-2 &= 2 \checkmark \end{aligned}$$

The figure below suggests that the sum of the angles of a pentagon is $3 \times 180^\circ = 540^\circ$.

If the pentagon is equiangular, then each angle is $540^\circ / 5 = 108^\circ$.

46. What is the sum of the angles of a hexagon

$$4(180^\circ) = 720^\circ$$

47. If the hexagon is equiangular, how large is each angle?

$$720^\circ / 6 = 120^\circ$$

48. What is the sum of the angles of an octagon?

$$6(180^\circ) = 1080^\circ$$

49. If the octagon is equiangular, how large is each angle?

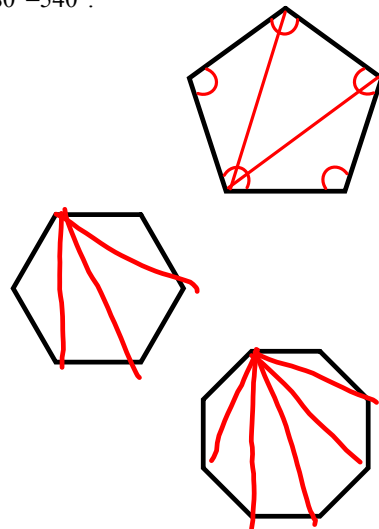
$$1080^\circ / 8 = 135^\circ$$

50. What, in terms of n , is the sum of the angles of an n -gon?

$$(n-2)180^\circ$$

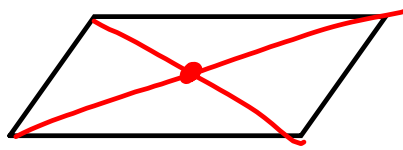
51. If the n -gon is equiangular, how large is each angle in terms of n ?

$$\frac{(n-2)180^\circ}{n}$$



7.2 – Parallelograms and Point Symmetry

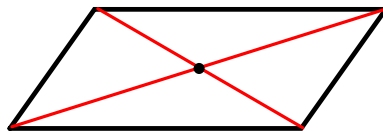
Def: A parallelogram is a quadrilateral whose opposite sides are parallel.



A figure has point symmetry if it looks exactly the same when it is rotated about a point.

Def: Two points are symmetric with respect to a point iff it is the midpoint of the line segment joining them.

Parallelograms have point symmetry about the point in which their diagonals intersect.



Theorem 25: The opposite sides and angles of a parallelogram are equal.

Given: ABCD is a parallelogram.

Prove: $AB=DC$, $AD=BC$, $\angle A=\angle C$, and $\angle B=\angle D$.

Proof: ✓

1. $AB \parallel DC$ & $AD \parallel BC$

2. $\angle 4 = \angle C$ $\angle 6 = \angle A$
 $\angle 7 = \angle C$ $\angle 9 = \angle A$

$\angle 3 = \angle D$ $\angle 1 = \angle B$
 $\angle 12 = \angle D$ $\angle 10 = \angle B$

$\angle 2 = \angle 4$, $\angle 3 = \angle 5$
 $\angle 8 = \angle 10$, $\angle 9 = \angle 11$

3. $\angle 1 = \angle 3$ $\angle 4 = \angle 6$
 $\angle A = \angle 2$ $\angle B = \angle 5$
 $\angle 7 = \angle 9$ $\angle 10 = \angle 12$
 $\angle D = \angle 8$ $\angle C = \angle 11$

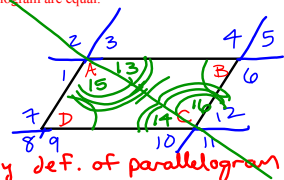
4. $\angle B = \angle D$
 $\angle A = \angle C$

5. Draw AC

6. $\angle 15 = \angle 16$
 $\angle 13 = \angle 14$

7. $\triangle ADC \cong \triangle CBA$

8. $AD = BC$
 $AB = DC$



by def. of parallelogram
 Parallel lines form equal corresponding angles

vertical angles are equal

subst. into $\angle 1 = \angle 3$

subst. into $\angle 4 = \angle 6$

2 points define a line

parallel lines form equal alternate interior \angle 's

AAS/ASA congruence

Corresponding parts of $\cong \Delta$'s are =

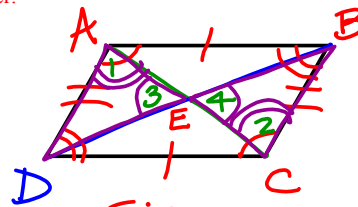
Theorem 26: The diagonals of a parallelogram bisect each other.

Given: ABCD is a parallelogram with diagonals AC and BD.

Prove: AC and BD bisect each other.

$(AE=EC \text{ \& } DE=EB)$

1. ABCD is a parallelogram
2. $AB \parallel DC$ & $AD \parallel BC$
3. $AB=DC$ & $AD=BC$
 $\angle A = \angle C$ & $\angle B = \angle D$
4. $\angle 3 = \angle 4$
5. $\angle 1 = \angle 2$
6. $\triangle ADE \cong \triangle BCE$
7. $AE=CE$ & $DE=BE$
8. E is a midpoint / bisector for the diagonals



Given
 def. of parallelogram
 parallelograms have equal opposite sides & angles
 Vertical \angle 's are =
 parallel lines form equal alternate interior \angle 's
 AAS congruence
 corresponding parts of congruent \triangle 's are =
 def. of midpoint