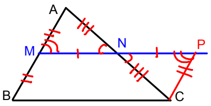
7.6 – The Midsegment Theorem

Def: A midsegment of a triangle is a line segment that connects the midpoints of two of its sides.

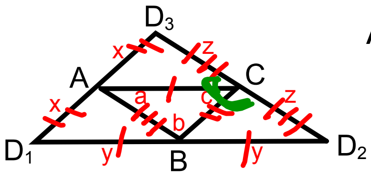
Theorem 37 : The Midsegment Theorem – A midsegment of a triangle is parallel to the third side and half as long.

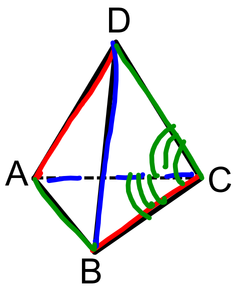
Given: MN is a midsegment of ΔABC.
Prove: MN||BC and MN=(1/2)BC.



1. Draw MN	2 pts define a line
2. choose point P on line MN so that MN=NP	Ruler Postulate
3. Draw CP	2 points define a line
4. AM=MB & AN=NC	M & N are midpoints of AB & AC respectively
5. ∠ANM=∠CNP	vertical ∠'s are equal
6. ΔANM≅ΔCNP	SAS congruence
7. AM=PC	corresponding parts of congruent triangles are equal
8. MB=PC	substitution
9. ∠AMN=∠CPN	corresponding parts of congruent Δ's are equal
10. ∠AMN and ∠BMN are supplementary	angles in a linear pair are supplementary
11. ∠AMN+∠BMN=180°	supplementary ∠'s sum to 180°
12. ∠BMN+∠CPN=180°	substitution
13. ∠BMN and ∠CPN are supplementary (jk, not needed)	angles that sum to 180° are supplementary
14. BM ∥ CP	supplementary interior angles on same side of transversal / equal (alternate interior angles) mean lines are parallel
15. BMPC is a parallelogram	quadrilateral w/ one pair of opposite sides that is both equal & parallel is a parallelogram
16. MP = BC	opposite sides of a parallelogram are equal
17. MN ∥ BC ✓	and parallel
18. MN+NP=MP	betweenness of points theorem
19. MN+MN=BC; 2MN=BC	substitutions (MN=NP & MP=BC)
20. MN = ½BC ✓	division

An edition of Euclid's Elements published in London in 1570 featured little paper models attached to the pages that could be folded up to form three-dimensional figures. One pattern consisted of a triangle and its three midsegments. It could be folded to form a tetrahedron, which is a polyhedron with four triangular faces.





38. What must be true about the four triangles in each figure?

congruent by SSS congruence

Two edges of a tetrahedron that do not intersect are called opposite edges; for example, AC and BD are opposite edges.

40. What do you notice about the opposite edges of the tetrahedron?

equal but not parallel (or even coplanar!)

41. What is the sum of the three angles at each vertex of the tetrahedron?

180°