Theorem 17: Equal corresponding angles mean that lines are parallel.
Corollary 1: Equal alternate interior angles mean that lines are parallel.
Corollary 2: Supplementary interior angles on the same side of a transversal mean that lines are parallel.
Corollary 3: In a plane, two lines perpendicular to a third line are parallel.

The Parallel Postulate – Through a point not on a line, there is exactly one line parallel to the given line.
Theorem 18: In a plane, two lines parallel to a third line are parallel to each other.

Theorem 19: Parallel lines form equal corresponding angles.
Corollary 1: Parallel lines form equal alternate interior angles.
Corollary 2: Parallel lines form supplementary interior angles on the same side of a transversal.
Corollary 3: In a plane, a line perpendicular to one of two parallel lines is also perpendicular to the other.

Theorem 20: The Triangle Sum Theorem – The sum of the angles of a triangle is 180°.
Corollary 1: If two angles of one triangle are equal to two angles of another triangle, the third angles are equal.
Corollary 2: The acute angles of a right triangle are complementary.
Corollary 3: Each angle of an equilateral triangle is 60°.

Theorem 21: An exterior angle of a triangle is equal to the sum of the remote interior angles.

Theorem 22: The AAS Theorem – If two angles and the side opposite one of them in one triangle are equal to the corresponding parts of another triangle, the triangles are congruent.

Theorem 23: The HL Theorem – If the hypotenuse and a leg of one right triangle are equal to the corresponding parts of another right triangle, the triangles are congruent.

SSS, SAS, AAS, ASA, HL

Def: A diagonal of a polygon is a line segment that connects any two nonconsecutive vertices.

Theorem 24: The sum of the angles of a quadrilateral is 360° .

Def: A rectangle is a quadrilateral each of whose angles is a right angle.

Corollary to Theorem 24: A quadrilateral is equiangular iff it is a rectangle.

In general, if a polygon has n sides, in terms of n,

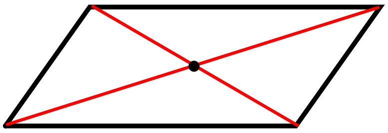
- n-3 diagonals can be drawn from one vertex
- these diagonals form n-2 triangles
- the sum of the angles of an n-gon is (n-2)*180°
- If the n-gon is equiangular, each angle measures (n-2)*180°/n

Def: A parallelogram is a quadrilateral whose opposite sides are parallel.

A figure has point symmetry if it looks exactly the same when it is rotated about a point.

Def: Two points are symmetric with respect to a point iff it is the midpoint of the line segment joining them.

Parallelograms have point symmetry about the point in which their diagonals intersect.



Theorem 25: The opposite sides and angles of a parallelogram are equal.
Theorem 26: The diagonals of a parallelogram bisect each other.
Theorem 27: A quadrilateral is a parallelogram, if its opposite sides are equal.
Theorem 28: A quadrilateral is a parallelogram if its opposite angles are equal.
Theorem 29: A quadrilateral is a parallelogram if two opposite sides are both parallel and equal.

Theorem 30: A quadrilateral is a parallelogram if its diagonals bisect each other.

7.4 – Rectangles, Rhombuses, and Squares

Def: A square is a quadrilateral all of whose sides and angles are equal.

Every square is a rhombus.

Def: A rhombus is a quadrilateral all of whose sides are equal.

Theorem 31: All rectangles are parallelograms.

Theorem 32: All rhombuses are parallelograms.

Theorem 33: The diagonals of a rectangle are equal.

Theorem 34: The diagonals of a rhombus are perpendicular.

Def: A trapezoid is a quadrilateral that has exactly one pair of parallel sides.

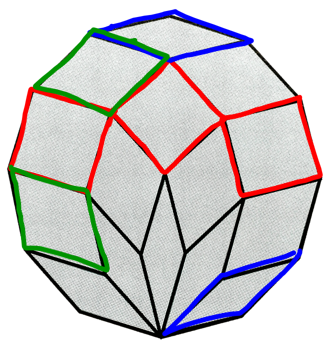
The parallel sides are called the bases of the trapezoid, and the non-parallel sides are called its legs. The pairs of angles that include each base are called base angles.

Def: An isosceles trapezoid is a trapezoid whose legs are equal.

Theorem 35: The base angles of an isosceles trapezoid are equal.
Theorem 36: The diagonals of an isosceles trapezoid are equal.
If a quadrilateral is a trapezoid, then its diagonals cannot bisect each other.

Def: A midsegment of a triangle is a line segment that connects the midpoints of two of its sides.
Theorem 37: The Midsegment Theorem – A midsegment of a triangle is parallel to the third side and half as long.

Regular dodecagon



1. How many sides does a dodecagon have?

12 sides

A regular polygon is one that is equilateral and equiangular.

2. How many regular quadrilaterals do there seem to be in the figure?

15

3. What is a regular quadrilateral called?

Square

4. How many rectangles do there seem to be in the figure?

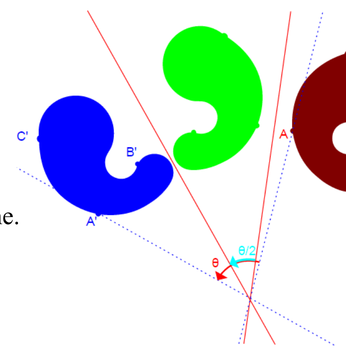
3

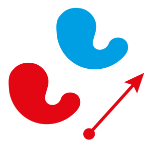
5. How many rhombuses are in the figure?

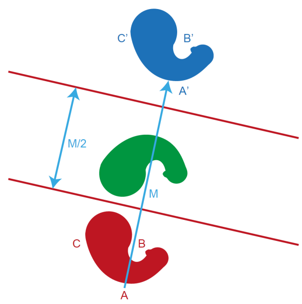
15

6. How many different shapes of rhombuses does the figure seem to contain? 3

8.1 – Transformations







Def: A transformation is a one-to-one correspondence between two sets of points.

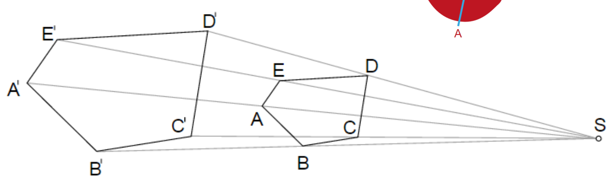
A translation slides an object a certain distance without turning it.

A reflection flips an object over a mirror line.

A rotation turns an object a certain number of degrees about a fixed point.

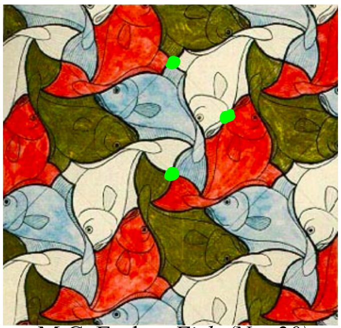
A dilation enlarges or reduces the size of an object.

Def: an isometry is a transformation that preserves distance and angle measure.



Translations, reflections, and rotations

are all examples of isometries, but dilations are not.



M.C. Escher. *Fish (No. 20)*

What type of translation seems to relate

1. Two fish of the same color?

   translation

2. A pair of red and white fish?

   rotation

3. A pair of blue and white fish?

   rotation

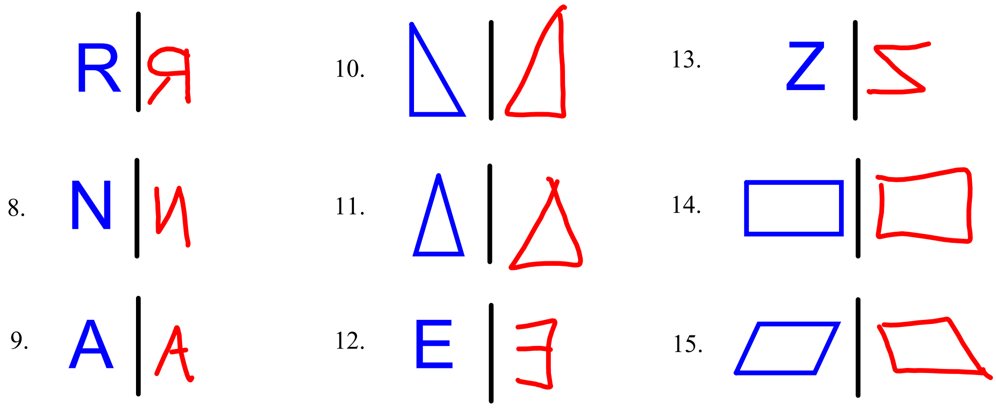
Are there any pairs of fish in the figure for which one fish of the pair seems to be

4. A dilation of the other?

   no

5. A reflection of the other? no

Complete the figures by including the reflection image of the object through the mirror line.



16. Which figures look the same as their mirror images?

#9, 11, 14

17. What is it about these figures that causes them and their mirror images to look the same?

8.2 – Reflections

Def: The reflection of point P through line l is P itself if P lines on l. Otherwise, it is the point P' such that l is the perpendicular bisector of PP'.

Construction 8: To reflect a point through a line.

Def: A translation is the composite of two successive reflections through parallel lines.

The distance between a point of the original figure and its translation image is called the *magnitude* of the translation.

Def: A rotation is the composite of two successive reflections through intersecting lines. The point in which the lines intersect is the *center* of rotation, and the measure of the angle through which a point of the original figure turns to coincide with its rotation image is called the *magnitude* of the rotation.

8.2 – Reflections

Def: The reflection of point P through line l is P itself if P lines on l. Otherwise, it is the point P' such that l is the perpendicular bisector of PP'.

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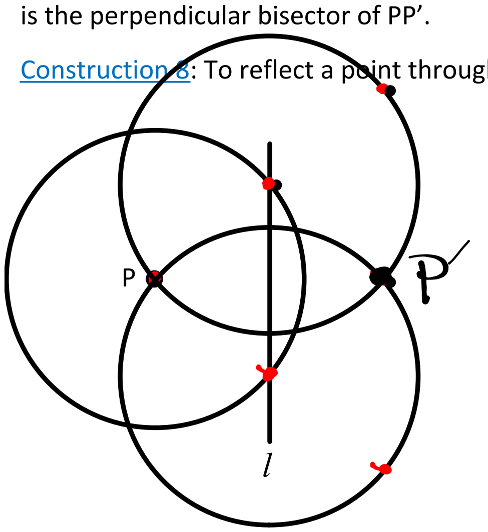
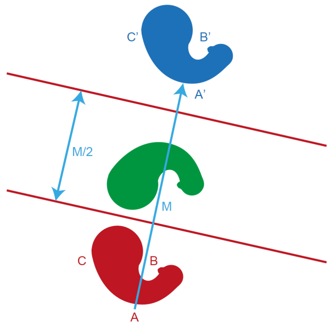
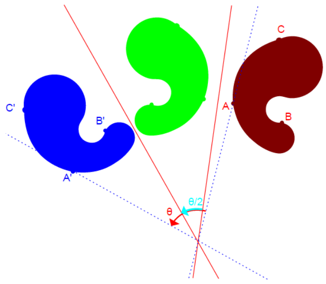


Def: A translation is the composite of two successive reflections through parallel lines.

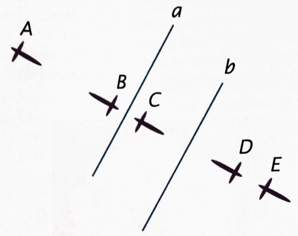
The distance between a point of the original figure and its translation image is called the *magnitude* of the translation.





Def: A rotation is the composite of two successive reflections through intersecting lines.
The point in which the lines intersect is the *center* of rotation, and the measure of the angle through which a point of the original figure turns to coincide with its rotation image is called the *magnitude* of the rotation.

In the figure below, $a \parallel b$ and birds A, B, D, and E are reflection images of bird C through either or both of the lines.



Which bird is the reflection image of

31. bird C through a?

32. bird B through b?

33. bird C through b?

34. bird D through a?

Which bird is the image of bird C as a result of successive reflections through

35. a and b?

36. b and a?

37. What transformation do exercises 35 and 36 illustrate?

31. B

32. E

33. D

34. A

35. E

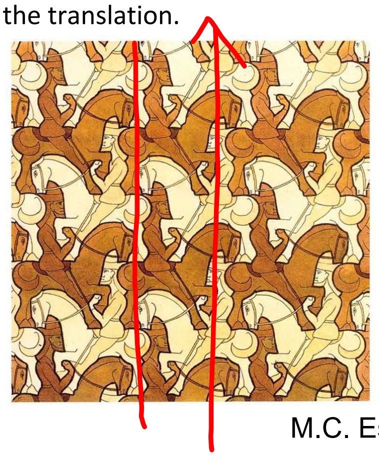
36. A

translations

8.3 - Isometries and Congruence

Def: Two figures are congruent if there is an isometry such that one figure is the image of the other.

Def: A glide reflection is the composite of a translation and a reflection in a line parallel to the direction of the translation.



M.C. Escher

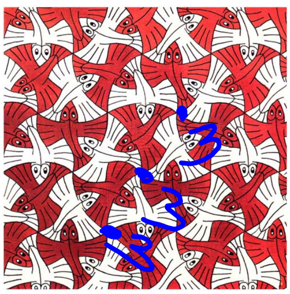
Koloman Moser

8.4 - Transformations and Symmetry

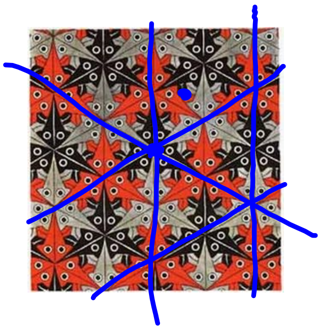
Def: A figure has rotation symmetry with respect to a point iff it coincides with its rotation image through less that 360° about the point.

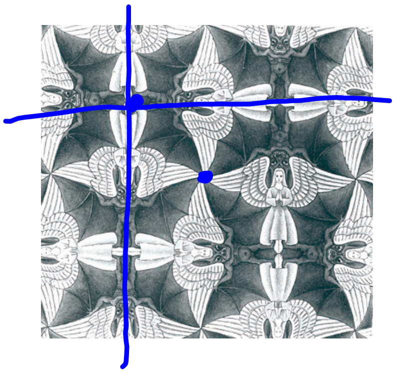
A figure is said to have n-fold rotation symmetry iff the smallest angle through which it can be turned to look exactly the same is 360°/n.

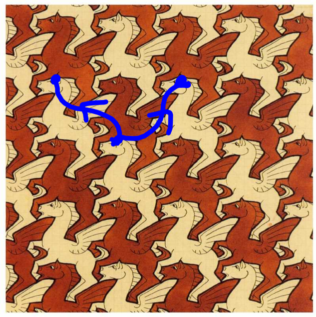
colored - 2
ignoring
4



Def: A figure has reflection (line) symmetry with respect to a line iff it coincides with its reflection image through the line. The line is sometimes called the axis of symmetry.







Def: A pattern has translation symmetry iff it coincides with a translation image.