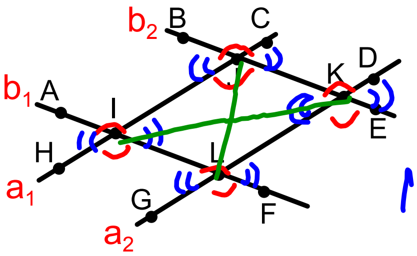
Due Wed. 27 Jan:

- Ch 8 Review, pp.326-327, #7-29

Due Wed. 03 Feb:

- Ch 9 Review, pp.
- Midterm Review, pp. 330-336, #1-125

Test #4 - Thurs. 04 Feb



Given $a_1 || a_2$ and $b_1 || b_2$, what else do we know?

$IL = JK$; $IJ = LK$

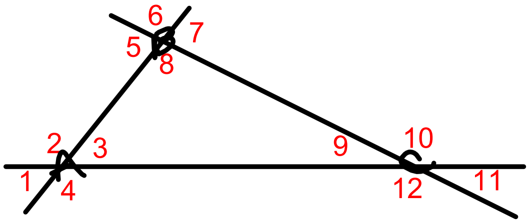
$180° = \angle JKL + \angle KLI = \angle LIJ + \angle IJK$
$= \angle IJK + \angle JKL =$
$= \angle JKL + \angle LIJ$

IJKL is a parallelogram

$\angle AIC = \angle ALD = \angle HIF = \angle GLF = \angle LKE = \angle JKD =$
$= \angle IJK = \angle BJC$

(('s are all =

IK & JL bisect each other



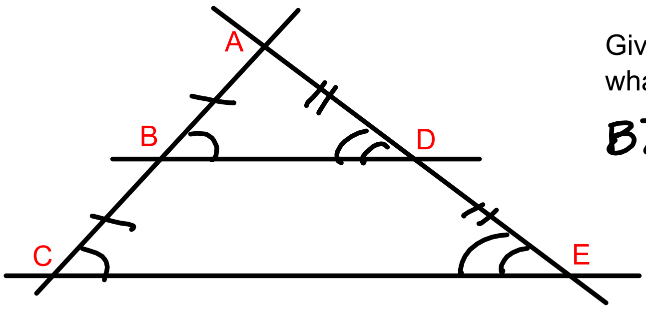
except = vertical ∠'s, what else do we know?

$\angle 2 = \angle 8 + \angle 9 = \angle 4$
$\angle 10 = \angle 3 + \angle 8 = \angle 12$
$\angle 5 = \angle 3 + \angle 9 = \angle 7$

exterior angles are equal to the sum of both remote interior angles

$\angle 3 + \angle 8 + \angle 9 = 180^\circ$

$AB + BC > AC$ Triangle Inequality Theorem



Given AB=BC and AD=DE,
what else do we know?

BD is a midsegment

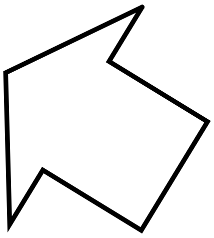
$BD = \frac{1}{2} CE$

$BD \parallel CE$

$AB + BC = AC$ Betweenness of points Theorem

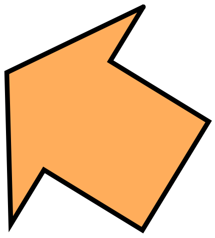
$\angle ABC = \angle BCE$
$\angle ADB = \angle DEC$

9.1 - Area



The black line is the polygon.

The region bounded by that polygon is a polygonal region.



When we find the area of a polygon, we are actually finding the area of the polygonal region bounded by that polygon.

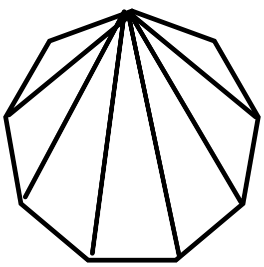
Postulate 8 - The Area Postulate

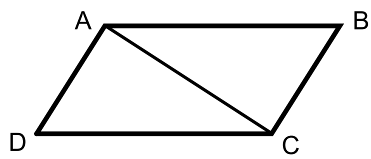
Every polygonal region has a positive number called its area such that

(1) congruent triangles have equal areas $\quad \alpha \triangle ABC = \alpha \triangle CDA$

(2) the area of a polygonal region is equal to the sum of the areas of its nonoverlapping parts

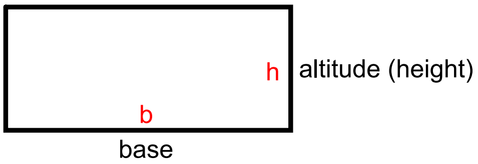
$$\alpha ABCD = \alpha \triangle ABC + \alpha \triangle CDA$$





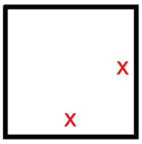
9.2 - Squares and Rectangles

Postulate 9 - The area of a rectangle is the product of its base and altitude



$A_{rect} = bh$

Corollary to Postulate 9 - The area of a square is the square of its side



$A_{sq} = x^2$

oops. not a square

To divide a square into smaller squares each having a different area was once thought to be impossible. The figure seems to show a solution.

Given that the areas of squares C and D are 64 and 81 square units respectively, find the areas of the other squares.

$\alpha C = 64$ $\alpha B = 15^2 = 225$

$\alpha D = 81$ $\alpha G = 4^2 = 16$

$\alpha I = 1^2 = 1$ $\alpha F = 14^2 = 196$

$\alpha E = 10^2 = 100$ $\alpha A = 18^2 = 324$

$\alpha H = 7^2 = 49$

$$18^2=(10+8)^2$$

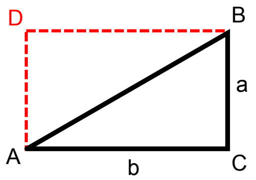
$$= 10^2+2\cdot 10\cdot 8+8^2$$

$$= 100+160+64$$

$$= 324$$

9.3 - Triangles

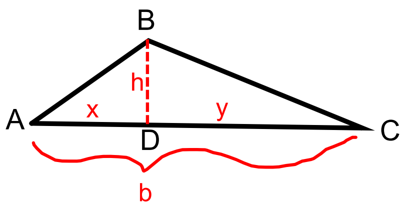
Theorem 38 - The area of a right triangle is half the product of its legs.



Given: Right $\triangle ABC$ with legs a and b

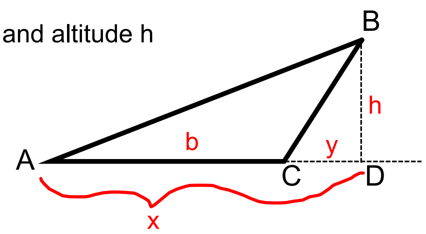
Prove: $\alpha\triangle ABC = \frac{1}{2}ba$

Theorem 39 - The area of a triangle is half the product of any base and corresponding altitude.

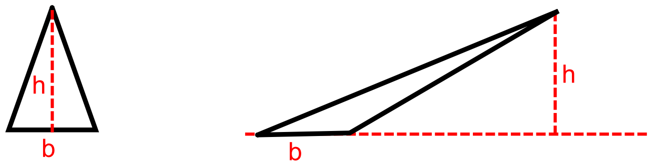


Given: $\triangle ABC$ with base b and altitude h

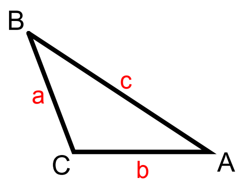
Prove: $\alpha\triangle ABC = \frac{1}{2}bh$



Corollary to Theorem 39 - Triangles with equal bases and equal altitudes have equal areas.



Heron's Theorem



The area of a triangle with sides a, b, and c is $\sqrt{s(s-a)(s-b)(s-c)}$ where s is half of the triangle's perimeter.

$s = \frac{1}{2}(a+b+c)$

Suppose there are three triangles with the following sides:

Triangle 1: 5, 5, and 6. $s=8,\ A=\sqrt{8(8-5)(8-5)(8-6)} = \sqrt{8\,3\,3\,2} = \sqrt{16}\cdot\sqrt{9} = 12$

Triangle 2: 5, 5, and 8. $s=9,\ A=\sqrt{9(9-5)(9-5)(9-8)} = \sqrt{9}\cdot\sqrt{16} = 3\cdot 4 = 12$

Triangle 3: 5, 5, and 10. $s=10,\ A=\sqrt{10(10-5)(10-5)(10-10)} = 0$

1. Which triangle do you think has the greatest area? $\triangle 3$

2. Use Heron's Theorem to find the area of each triangle.

3. One of the "triangles" isn't really a triangle. Which one and why not?

#3 fails the $\triangle$ inequality

Now, suppose there are two triangles with the following sides:

Triangle 4: 4, 6, and 8.

Triangle 5: 400, 600, and 1000.

4. Which do you think has the greater area? 4 (#5 is not a $\triangle$)

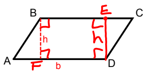
5. Use Heron's Theorem to find it.

$s = \frac{4+6+8}{2} = 9$

$A = \sqrt{9(9-4)(9-6)(9-8)} = \sqrt{9\cdot5\cdot3\cdot1} = \boxed{3\sqrt{15}}$

9.4 - Parallelograms and Trapezoids

Theorem 40 - The area of a parallelogram is the product of any base and corresponding altitude.



Given: ABCD is a parallelogram with altitude h corresponding to base b
Prove: $\alpha ABCD = bh$

1. Draw $\overline{DE}$ so that $\overline{DE} \perp \overline{BC}$ — there is a unique perpendicular line through a pt not on a given line
2. $\overline{BE} \perp \overline{BF}$, $\overline{BE} \perp \overline{ED}$ — a line perpendicular to one of a pair of parallel lines is also perpendicular to the other
3. $\angle FBE$, $\angle BED$, $\angle EDF$, $\angle FDB$ are right $\angle$'s — perpendicular lines form right $\angle$'s
4. BEDF is a rectangle, $\triangle ABF$ & $\triangle CED$ are right $\triangle$'s — def's of rect & right $\triangle$
5. $AF + FD = b$, $BE + EC = BC$ — Betweenness of Pts Thm
6. $ED = h$, $BE = FD$ — opposite sides of a rectangle, which is a parallelogram, are equal
7. $\alpha BEDF = (FD) \cdot h$ — α rect $= bh$
8. $\alpha \triangle ABF = \frac{1}{2}(AF)h$, $\alpha \triangle CED = \frac{1}{2}(EC)h$ — α right $\triangle = \frac{1}{2}bh$
9. $\alpha ABCD = \alpha BEDF + \alpha \triangle ABF + \alpha \triangle CED$ — Area Postulate
10. $\alpha ABCD = (FD)h + \frac{1}{2}(AF)h + \frac{1}{2}(EC)h$
11. $AF = EC$ — a bunch of substitutions
12. $\alpha ABCD = (FD)h + \frac{1}{2}(AF)h + \frac{1}{2}(AF)h$

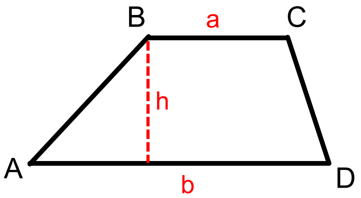
$= (FD)h + (AF)h$

$= h(FD + AF)$ — dist.

$= h(AD)$

$\alpha ABCD = bh$

Theorem 41 - The area of a trapezoid is half the product of its altitude and the sum of its bases.



Given: ABCD is a trapezoid w/ bases $\underset{a}{BC}$ & $\underset{b}{AD}$, altitude h
Prove: $\alpha ABCD = \frac{1}{2}(a + b) \cdot h$