Homework - due Fri 12 Feb
Ch 10 Review, pp. 421-424 #1-62

Final Exam: Wed 17 Feb 9-11am

Def: The ratio of the number a to the number b is the number a/b.
A proportion is an equality between ratios. a/b=c/d

a, b, c, and d are called the *first, second, third, and fourth terms*.
The second and third terms, b and c, are called the means.
The first and fourth terms, a and d, are called the extremes.

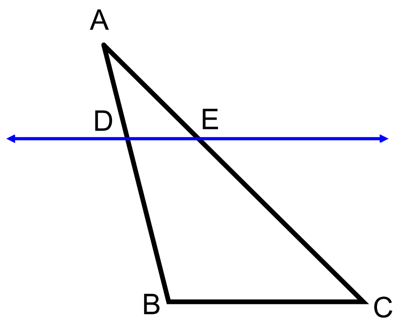
The product of the means is equal to the product of the extremes.
If a/b=c/d, then ad=bc.

Def: The number b is the geometric mean between the numbers a and c if a, b, and c are positive and a/b=b/c. ↳betw. x&y $=\sqrt{xy}$

Def: Two triangles are similar iff there is a correspondence between their vertices such that their corresponding sides are proportional and their corresponding angles are equal.

10.3 - The Side-Splitter Theorem

Theorem 44 - The Side-Splitter Theorem
If a line parallel to one side of a triangle intersects the other two sides in different points, it divides the sides in the same ratio, that is, if in triangle ABC, DE||BC, then AD/DB=AE/EC.



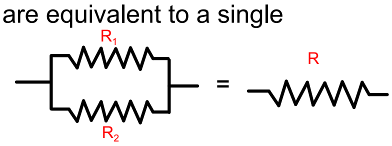
Corollary to the Side-Splitter Theorem:
If a line parallel to one side of a triangle intersects the other two sides in different points, it cuts off segments proprtional to the sides, that is, AD/AB=AE/AC and DB/AB=EC/AC

10.4 - AA Similarity

Theorem 45 - The AA Theorem - If two angles of one triangle are equal to two angles of another triangle, the triangles are similar.

Corollary to the AA Theorem - Two triangles similar to a third triangle are similar to each other.

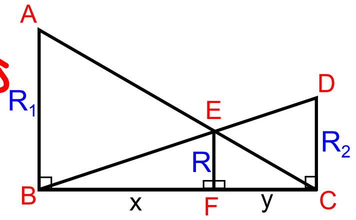
Electricians know that if two resistances R_1 and R_2 are "in parallel," they are equivalent to a single resistance R, where $R=(R_1R_2)/(R_1+R_2)$.



Prove that the figure below illustrates this equation by giving a reason for each of the following statements.

25. $\Delta EFC \sim \Delta ABC$ and $\Delta EFB \sim \Delta DCB$

$\angle ABC = \angle EFC$
$\angle ACB = \angle ECF$ } AA Similarity



26. $R/R_1=y/(x+y)$ and $R/R_2=x/(x+y)$

Corresponding sides of similar triangles are proportional

27. $R/R_1+R/R_2=y/(x+y)+x/(x+y)=(y+x)/(x+y)=1$

addition

$$\frac{R}{R_1}+\frac{R}{R_2}=1$$

28. $RR_2+RR_1=R_1R_2$

multiplication

$$R_1R_2\left(\frac{R}{R_1}+\frac{R}{R_2}\right)=1\cdot R_1R_2$$

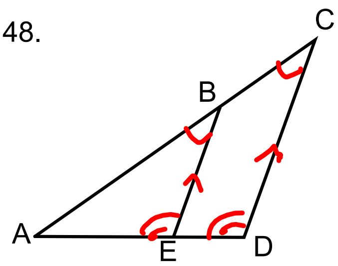
29. $R(R_2+R_1)=R_1R_2$

distributive prop.

$$\cancel{R_1}R_2\cdot\frac{R}{\cancel{R_1}}+R_1\cancel{R_2}\cdot\frac{R}{\cancel{R_2}}$$

30. $R=(R_1R_2)/(R_1+R_2)$ Division

48.



Given: ΔACD with $BE||CD$
Prove: $\Delta ABE \sim \Delta ACD$

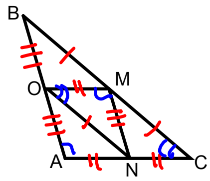
1. $\angle ABE = \angle ACD$
$\angle AEB = \angle ADC$

Parallel lines form equal corresponding angles

2. $\Delta ABE \sim \Delta ACD$

AA similarity theorem

50.



Given: ΔABC with midsegments MN, MO, and NO
Prove: ΔMNO~ΔABC

1. BM = MC, AN = NC, AO = OB — midpoints divide line segments into 2 equal parts

2. MO ∥ AC, MN ∥ AB, NO ∥ BC; MO = ½AC, MN = ½AB, NO = ½BC
2.5 AC = 2MO, AB = 2MN, BC = 2NO — Midsegment Theorem for Triangles; multiplication

3. AC = AN + NC = AN + AN = 2AN — Betweenness of Points Theorem

4. AB = AO + OB = AO + AO = 2AO
BC = BM + MC = BM + BM = 2BM — Substitution & Simplification

5. 2MO = 2AN, 2MN = 2AO, 2NO = 2BM; MO = AN, MN = AO, NO = BM — Substitution & division

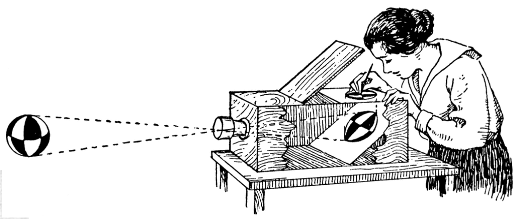
6. ΔAON ≅ ΔMNO ≅ ΔNMC — SSS Congruence

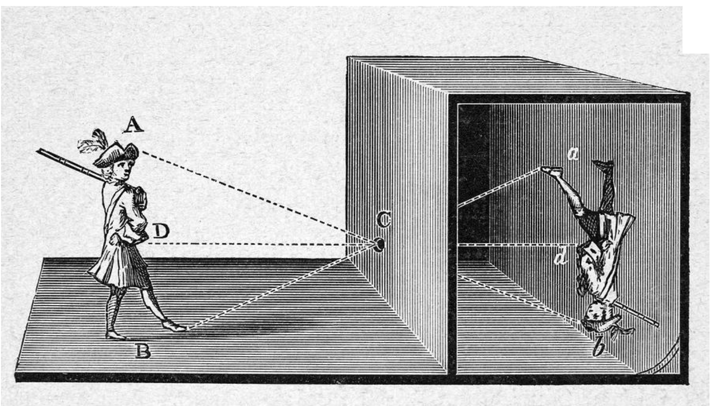
7. ∠A = ∠OMN, ∠C = ∠MON — correspond

8. ΔMNO ~ ΔABC — AA Similarity Theorem

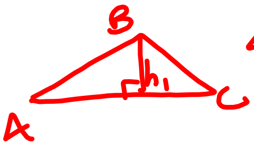
10.5 - Proportions and Dilations

Camera Obscura





Theorem 46 - Corresponding altitudes of similar triangles have the same ratio as that of the corresponding sides.



$\triangle ABC \sim \triangle DEF$

$$\frac{AB}{DE} = \frac{h_1}{h_2}$$

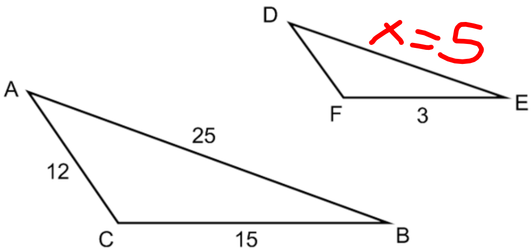
Theorem 47 - The ratio of the perimeters of two similar polygons is equal to the ratio of the corresponding sides.

$$\frac{AB}{DE} = \frac{AB+BC+AC}{DE+EF+FD} = \frac{P\triangle ABC}{P\triangle DEF}$$

Theorem 48 - The ratio of the areas of two similar polygons is equal to the square of the ratio of the corresponding sides.

$$\frac{\alpha\triangle ABC}{\alpha\triangle DEF} = \frac{(AB)^2}{(DE)^2}$$

Length of side DE

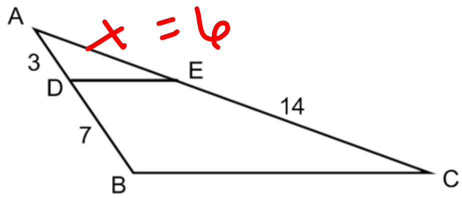


$x = 5$

$$\frac{x}{25} = \frac{3}{15}$$

$$x = \frac{3\cdot\cancel{25}\,5}{\cancel{15}\,\cancel{3}} = \boxed{5}$$

Length of segment AE



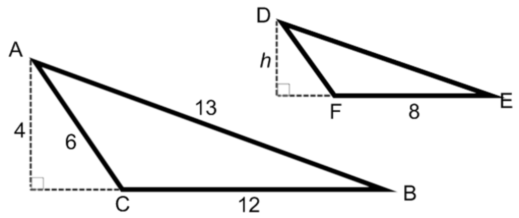
$x = 6$

$$\frac{x}{14} = \frac{3}{7}$$

$$x = \frac{3\cdot\cancel{14}\,2}{\cancel{7}}$$

$$x = \boxed{6}$$

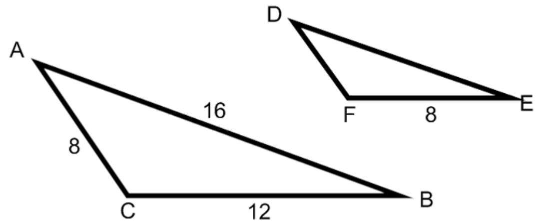
Altitude h of ΔDEF



$$\frac{h}{4} = \frac{8}{12}$$

$$h = \frac{8\cdot\cancel{4}}{\cancel{12}_{3}} = \boxed{\frac{8}{3}}$$

Perimeter of ΔDEF



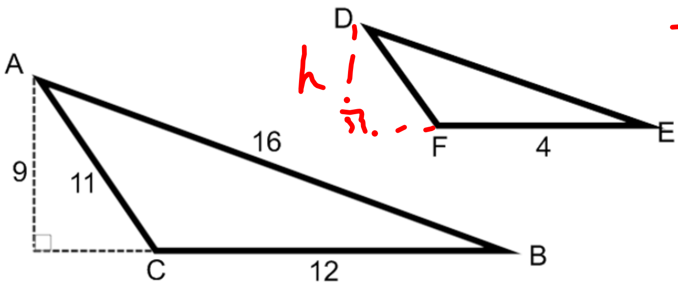
$$\frac{p\triangle DEF}{p\triangle ABC} = \frac{FE}{BC}$$

$$\frac{p\triangle DEF}{8+12+16} = \frac{8}{12}$$

$$p\triangle DEF = \frac{8}{\cancel{12}}\cdot\cancel{36}^{3}$$

$$= \boxed{24}$$

Area of ΔDEF



$$\frac{h}{9} = \frac{4}{12} \quad h = \frac{\cancel{4}}{\cancel{12}_{3}}\cdot\cancel{9}^{3} = 3$$

$$a\triangle DEF = \frac{1}{2}(4)(3)$$

$$= 6$$

$$\frac{a\triangle DEF}{a\triangle ABC} = \frac{(FE)^2}{(CB)^2}$$

$$\frac{a\triangle DEF}{\frac{1}{2}(12)(9)} = \frac{4^2}{12^2}$$

$$a\triangle DEF = \frac{\cancel{16}}{\cancel{3\cdot4\cdot3\cdot4}}\cdot 6\cdot\cancel{9} = \boxed{6}$$