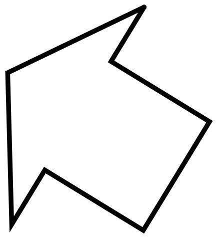
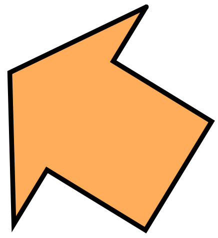
## 9.1 - Area



The black line is the polygon.

The region bounded by that polygon is a polygonal region.



When we find the area of a polygon, we are actually finding the area of the polygonal region bounded by that polygon.

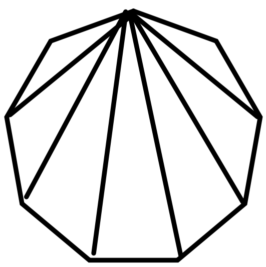
Postulate 8 - The Area Postulate

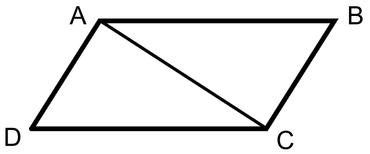
Every polygonal region has a positive number called its area such that

(1) congruent triangles have equal areas $\alpha \triangle ABC = \alpha \triangle CDA$

(2) the area of a polygonal region is equal to the sum of the areas of its nonoverlapping parts

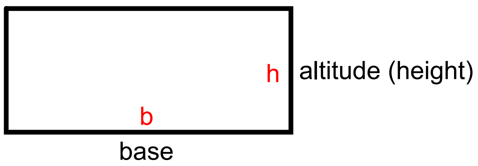
$$\alpha ABCD = \alpha \triangle ABC + \alpha \triangle CDA$$





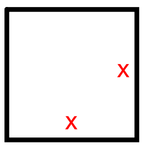
## 9.2 - Squares and Rectangles

Postulate 9 - The area of a rectangle is the product of its base and altitude

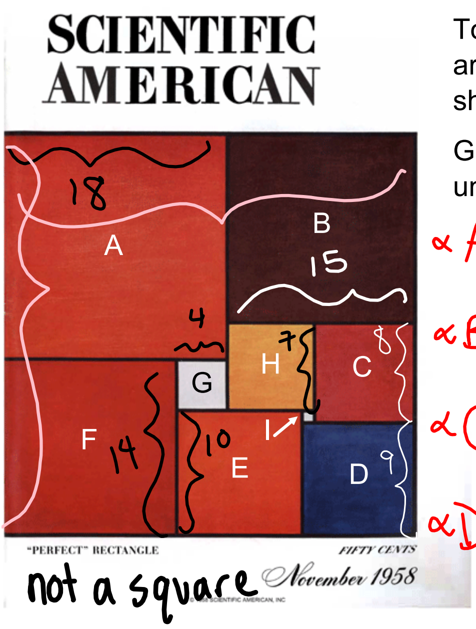



$$\alpha_{rect} = bh$$

Corollary to Postulate 9 - The area of a square is the square of its side




$$\alpha_{sq} = x^2$$




To divide a square into smaller squares each having a different area was once thought to be impossible. The figure seems to show a solution.

Given that the areas of squares C and D are 64 and 81 square units respectively, find the areas of the other squares.

$\alpha A = 18^2 = 324$

$\alpha B = 15^2 = 225$

$\alpha C = 64 = 8^2$

$\alpha D = 81 = 9^2$

$\alpha E = 10^2 = 100$

$\alpha F = 14^2 = 196$

$\alpha G = 4^2 = 16$

$\alpha H = 7^2 = 49$

$\alpha I = 1^2 = 1$

$$18^2=(10+8)^2=(10+8)(10+8)$$

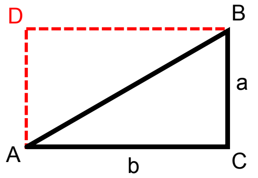
$$=100+\underbrace{80+80}_{160}+64$$

$$=260+64$$

$$=324$$

## 9.3 - Triangles

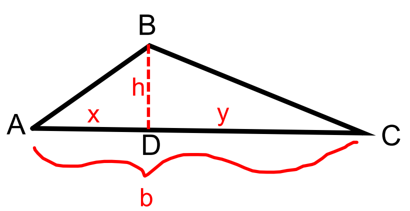
Theorem 38 - The area of a right triangle is half the product of its legs.




*Given:* Right $\triangle ABC$ with legs a and b

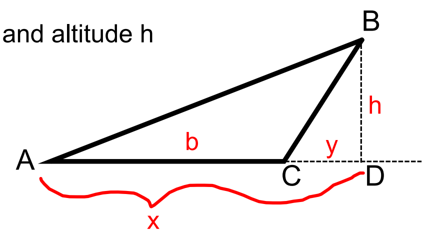
*Prove:* $\alpha \triangle ABC = \frac{1}{2}ba$

Theorem 39 - The area of a triangle is half the product of any base and corresponding altitude.

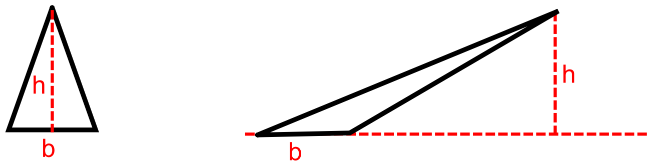



*Given:* $\triangle ABC$ with base b and altitude h

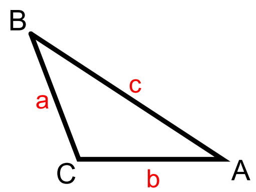
*Prove:* $\alpha \triangle ABC = \frac{1}{2}bh$



<u>Corollary to Theorem 39</u> - Triangles with equal bases and equal altitudes have equal areas.




### Heron's Theorem




The area of a triangle with sides a, b, and c is $\sqrt{s(s-a)(s-b)(s-c)}$ where s is half of the triangle's perimeter.

$S = \frac{1}{2}(a+b+c)$

Suppose there are three triangles with the following sides:

Triangle 1: 5, 5, and 6. $s = 8;\ \alpha = \sqrt{8(8-5)(8-5)(8-6)} = \sqrt{8 \cdot 3 \cdot 3 \cdot 2} = \sqrt{3^2 \cdot 4^2} = 3 \cdot 4 = 12$ ($8 \cdot 2 = 16$)

Triangle 2: 5, 5, and 8. $S = 9;\ \alpha = \sqrt{9(9-5)(9-5)(9-8)} = \sqrt{9 \cdot 4 \cdot 4} = \sqrt{3^2 \cdot 4^2} = 12$

Triangle 3: 5, 5, and 10.

$S = 10;\ \alpha = \sqrt{10(10-5)(10-5)(10-10)} = \sqrt{0} = 0$

1. Which triangle do you think has the greatest area? 3
2. Use Heron's Theorem to find the area of each triangle.
3. One of the "triangles" isn't really a triangle. Which one and why not? $\Delta$ 3 fails the Triangle Inequality

Now, suppose there are two triangles with the following sides:

Triangle 4: 4, 6, and 8.

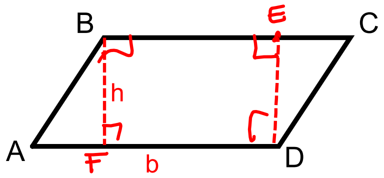
Triangle 5: 400, 600, and 1000.

4. Which do you think has the greater area? $\Delta 4$ (5 is not a $\Delta$)
5. Use Heron's Theorem to find it. $S = 9$

$\alpha = \sqrt{9(9-4)(9-6)(9-8)} = \sqrt{9 \cdot 5 \cdot 3 \cdot 1} = 3\sqrt{15}$

**9.4 - Parallelograms and Trapezoids**

Theorem 40 - The area of a parallelogram is the product of any base and corresponding altitude.




Given: ABCD is a parallelogram
h is an altitude corresponding to base $b = AD$

To prove: $\alpha$ ABCD $= bh$

Proof:

| | |
|---|---|
| 1. choose point E on BC so that we can draw DE perpendicular to BC | Protractor postulate & 2 points define a line |
| 2. $\angle BFD$ is a right $\angle$ | altitude h is perpendicular to base |
| 3. $h \perp BE$ & $DE \perp AD$ | if a line is perpendicular to one of a pair of $\parallel$ lines, it's $\perp$ to the othe |
| 4. $\angle FBE$ & $\angle ADE$ are right $\angle$'s | perpendicular lines meet at right angles |
| 5. BEDF is a rectangle | def. of rectangle |