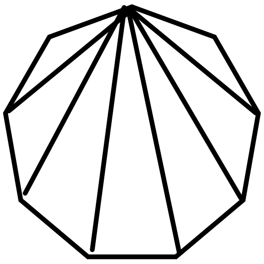
Postulate 8 - The Area Postulate

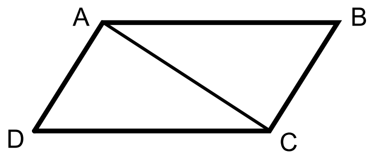
Every polygonal region has a positive number called its area such that

(1) congruent triangles have equal areas $\alpha \triangle ABC = \alpha \triangle CDA$

(2) the area of a polygonal region is equal to the sum of the areas of its nonoverlapping parts

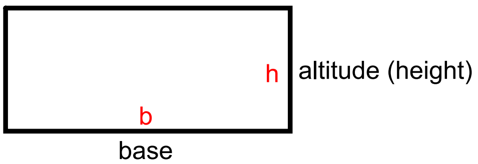
$$\alpha ABCD = \alpha \triangle ABC + \alpha \triangle CDA$$





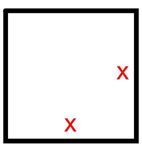
9.2 - Squares and Rectangles

Postulate 9 - The area of a rectangle is the product of its base and altitude



$$\alpha_{rect} = bh$$

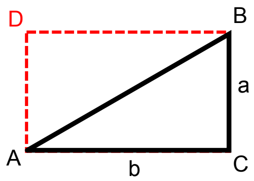
Corollary to Postulate 9 - The area of a square is the square of its side



$$\alpha_{sq} = x^2$$

9.3 - Triangles

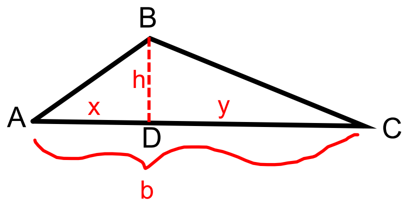
Theorem 38 - The area of a right triangle is half the product of its legs.



Given: Right $\triangle ABC$ with legs a and b

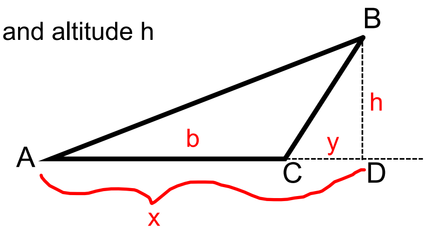
Prove: $\alpha \triangle ABC = \frac{1}{2}ba$

Theorem 39 - The area of a triangle is half the product of any base and corresponding altitude.

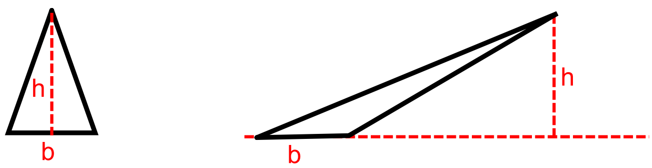


Given: $\triangle ABC$ with base b and altitude h

Prove: $\alpha \triangle ABC = \frac{1}{2}bh$



Corollary to Theorem 39 - Triangles with equal bases and equal altitudes have equal areas.

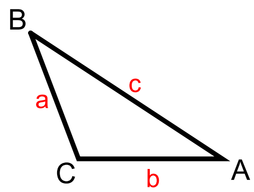


Heron's Theorem

The area of a triangle with sides a, b, and c is $\sqrt{s(s-a)(s-b)(s-c)}$

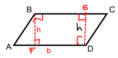
where s is half of the triangle's perimeter.

$S = \frac{1}{2}(a+b+c)$



9.4 - Parallelograms and Trapezoids

Theorem 40 - The area of a parallelogram is the product of any base and corresponding altitude.



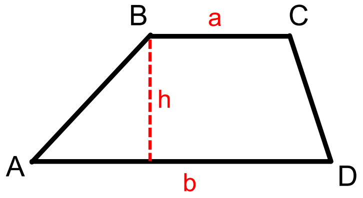
Given: ABCD is a parallelogram
h is an altitude corresbonding to base $b = AD$

To prove: $\alpha ABCD = bh$

Proof:

1. choose point E on BC so that we can draw DE perpendicular to BC	Protractor postulate & 2 points define a line
2. $\angle BFD$ is a right $\angle$	altitude h is perpendicular to base
3. $h \perp BE$ & $DE \perp AD$	if a line is perpendicular to one of a pair of ∥ lines, it is ⊥ to the other
4. $\angle FBE$ & $\angle ADE$ are right $\angle$'s	perpendicular lines meet at right angles
5. BEDF is a rectangle	def. of rectangle
6. $\alpha ABCD = \alpha \triangle ABF + \alpha \triangle CDE + \alpha BEDF$	Area postulate
7. $\alpha \triangle ABF = \frac{1}{2}(AF)h$	area of $\triangle$ is $\frac{1}{2}bh$
8. $ED = h$; $BE = FD$	BEDF is a parallelogram & opposite sides of parallelograms are =
9. $\alpha \triangle CDE = \frac{1}{2}(EC)h$	area of $\triangle$ is $\frac{1}{2}bh$
10. $\alpha BEDF = (FD)h$	area of rectangle is bh
11. $\alpha ABCD = \frac{1}{2}(AF)h + \frac{1}{2}(EC)h + (FD)h = h\left(\frac{1}{2}AF + \frac{1}{2}EC + FD\right)$	substitution 7, 9, & 10 into #6
12. $BC = AD$	opposite sides of a parallelogram are =
13. $BC = BE + EC$, $AD = AF + FD$	Betweenness of Points Theorem
14. $BE + EC = AF + FD$	substitution
15. $FD + EC = AF + FD$	Subst. #8 into 14
16. $EC = AF$	subtraction
17. $\alpha ABCD = h\left(\frac{1}{2}AF + \frac{1}{2}AF + FD\right) = h(AF + FD)$	Subst. #16 into 11
18. $\alpha ABCD = h(AD)$	substitution #13 into 17
$\alpha ABCD = bh$	Subst. w/ given $b = AD$

Theorem 41 - The area of a trapezoid is half the product of its altitude and the sum of its bases.

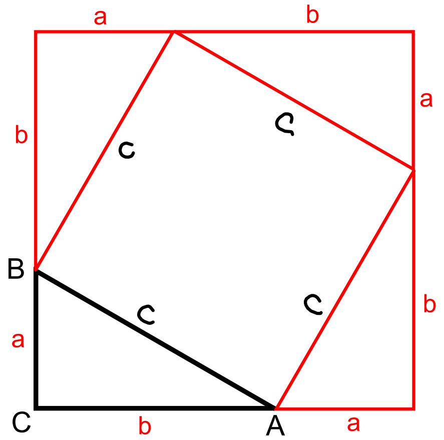


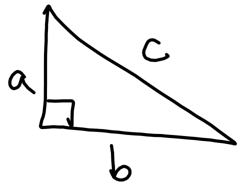
$$\alpha ABCD = \frac{1}{2}(a+b)h$$

$$= \frac{(a+b)h}{2}$$

9.5 - The Pythagorean Theorem

Theorem 42 (The Pythagorean Theorem) - The square of the hypotenuse of a right triangle is equal to the sum of the squares of its legs.





$$a^2+b^2=c^2$$

$$\text{big square} = (a+b)^2$$
$$= a^2+2ab+b^2$$
$$= 4\cdot\tfrac{1}{2}ab + c^2 = c^2+2ab$$
$$a^2+\cancel{2ab}+b^2 = c^2+\cancel{2ab}$$
$$a^2+b^2=c^2$$

9.1 a-d
9.3 a-b
9.5 a-g
9.10 a-d
9.13 a-d

9.15 a-d
9.16 a-c
9.17 a-c
9.24 a-b, d-e