Geometry - Area, cont. **February 28, 2018**

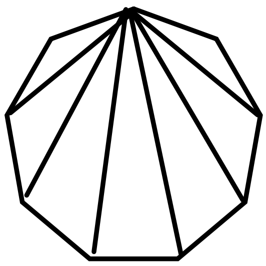
Postulate 8 - The Area Postulate

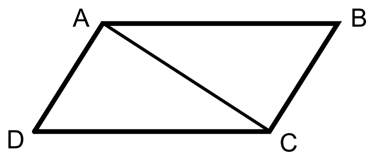
Every polygonal region has a positive number called its area such that

(1) congruent triangles have equal areas $\alpha\triangle ABC = \alpha\triangle CDA$

(2) the area of a polygonal region is equal to the sum of the areas of its nonoverlapping parts

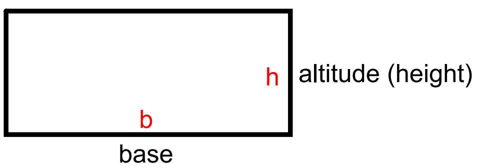
$$\alpha ABCD = \alpha\triangle ABC + \alpha\triangle CDA$$





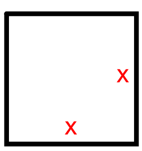
9.2 - Squares and Rectangles

Postulate 9 - The area of a rectangle is the product of its base and altitude



$\alpha_{rect} = bh$

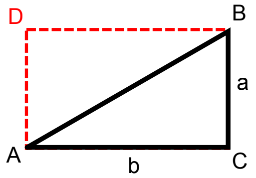
Corollary to Postulate 9 - The area of a square is the square of its side



$\alpha_{sq} = x^2$

9.3 - Triangles

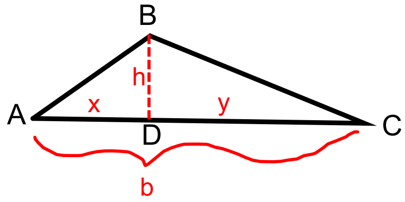
Theorem 38 - The area of a right triangle is half the product of its legs.



Given: Right $\triangle ABC$ with legs a and b

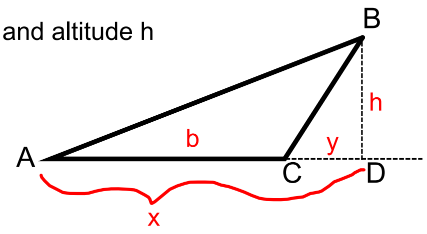
Prove: $\alpha \triangle ABC = \frac{1}{2}ba$

Theorem 39 - The area of a triangle is half the product of any base and corresponding altitude.

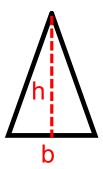


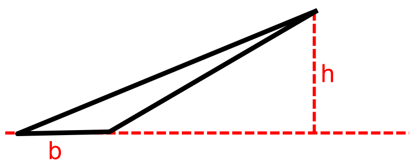
Given: $\triangle ABC$ with base b and altitude h

Prove: $\alpha \triangle ABC = \frac{1}{2}bh$



Corollary to Theorem 39 - Triangles with equal bases and equal altitudes have equal areas.



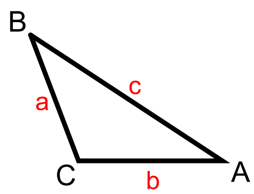


Heron's Theorem

The area of a triangle with sides a, b, and c is $\sqrt{s(s-a)(s-b)(s-c)}$

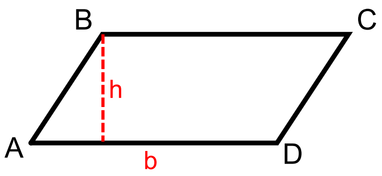
where s is half of the triangle's perimeter.

$S = \frac{1}{2}(a+b+c)$



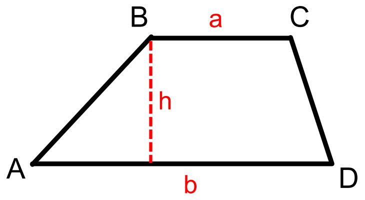
Geometry - Area, cont. **February 28, 2018**

Theorem 40 - The area of a parallelogram is the product of any base and corresponding altitude.



$\propto ABCD = bh$

Theorem 41 - The area of a trapezoid is half the product of its altitude and the sum of its bases.



$$\propto ABCD = \frac{1}{2}(a+b)h$$

$$= \frac{(a+b)h}{2}$$

9.5 - The Pythagorean Theorem

Theorem 42 (The Pythagorean Theorem) - The square of the hypotenuse of a right triangle is equal to the sum of the squares of its legs.

Theorem 43 (Converse of the Pythagorean Theorem) - If the square of one side of a triangle is equal to the sum of the squares of the other two sides, the triangle is a right triangle.

If a, then b

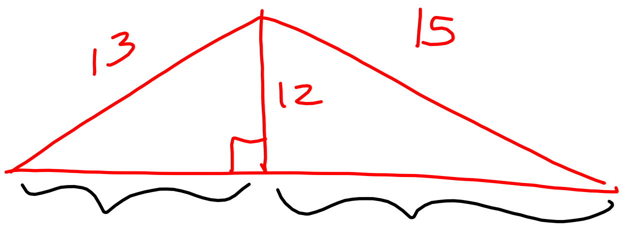
converse : If b, then a

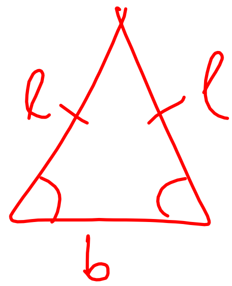
9.1 a-d
9.3 a-b
9.5 a-g
9.10 a-d
9.13 a-d

9.15 a-d
9.16 a-c
9.17 a-c
9.24 a-b, d-e

9.15

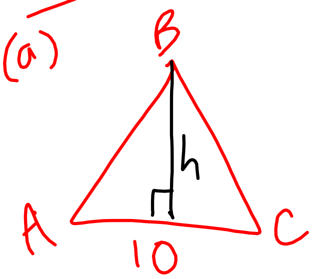
(a)



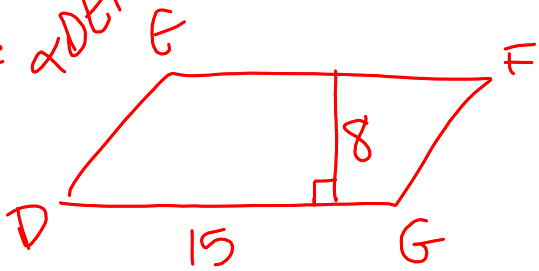


9.16

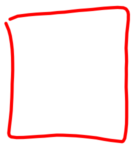
(a)



$\alpha ABC = \alpha DEFG$



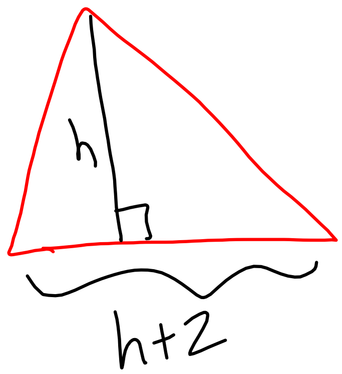
$$\frac{1}{2}(10)(h) = 15(8)$$



$1.8 = \frac{9}{5}$

$$\alpha = \left(\frac{9}{5}\right)^2$$

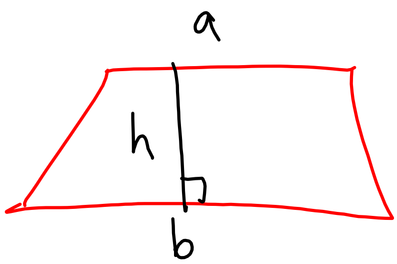
$$1.8 = 1\frac{8}{10} = \frac{18}{10} = \frac{9}{5}$$



$$24 = \frac{1}{2}h(h+2)$$

9.24

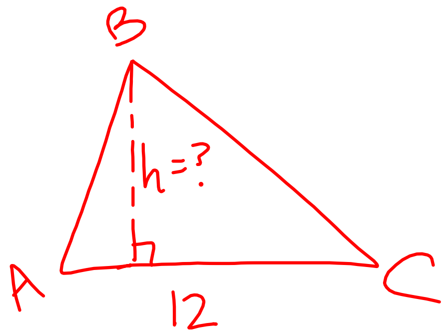
(b)

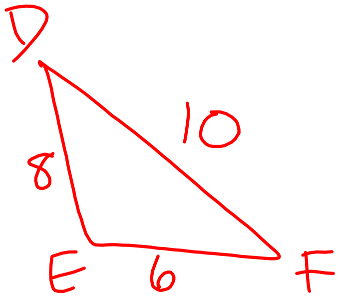


$$a+b = 2h \qquad \alpha = 49$$

$$\alpha = \frac{1}{2}(a+b)\,h$$

$$49 = \frac{1}{2}(2h)\,h$$





$$a\triangle ABC = a\triangle DEF = 24$$

$$\frac{1}{2}(12)(h) = 24$$