Rational Numbers
- the set of all numbers that can be written as fractions
- allow us to divide and define multiplicative inverses
- the set of all terminating and repeating decimals

$\mathbb{Q} = \left\{\frac{p}{q} | p, q \in \mathbb{Z}\right\}$ (such that)

Irrational Numbers
- the set of all non-terminating, non-repeating decimals
- includes $\pi, \sqrt{2},$ etc.

Real Numbers
- the set of rationals together with the irrationals
- any number than can be written as a decimal

$\mathbb{R}$

$-\infty \longleftrightarrow \infty$

1.1 Introduction to Real Numbers

Set – collection of objects

Element – an object in a set $a \in \{a,b,c\}$

Natural numbers = counting numbers
- if we add two natural numbers together, we get a natural number

{1,2,3,4,...} $= \mathbb{N}$

Prime number = only factors are 1 and itself

{2,3,5,7,11...}

Composite number = has factors other than 1 and itself

{4,6,8,9,10,...}

Integers = set of all positive and negative whole numbers, plus 0
- integers allow us to subtract (same as adding a negative)

{...,-3,-2,-1,0,1,2,3,...} $= \mathbb{Z}$

$\{0,1,2,3,\ldots\}$ whole numbers

Methods of Writing a Set

Roster Method – list

{1, 2, 3,…); {…-2,-1,0,1,2,…}; {1/2, 3.74, π}; {a, b, c}

Set-Builder Notation

{variable(s) | condition(s) on the variables}

{p/q | p,q ∈ **Z**}

"the set of fractions p/q such that p and q are elements of the set of integers"

{x | x ≥ 2}

"the set of numbers x such that x is greater than or equal to 2"

Interval Notation

$b \geq x \geq a$

closed interval - includes both endpoints

$[a,b]$ $\{x \mid a \leq x \leq b\}$

open interval - does not include either endpoint

(a,b) $\{x \mid a < x < b\}$

half-open or half-closed or "clopen" interval - includes one endpoint only

$(a,b]$ $\{x \mid a < x \leq b\}$

$[a,b)$ $\{x \mid a \leq x < b\}$

$\{\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}\}$

$\mathbb{R} = (-\infty, \infty)$ *infinity is not a number, so it is never included

$\emptyset$ = the empty set = the set containing no elements

$\{\emptyset\}$ = the set containing the empty set as an element

Union, Intersection, and Relative Complement

Union = the set of all of the elements from any set

$A \cup B$ (A∪B) = the set of elements from either set A or set B

Intersection = the set of all elements that occur in each set; overlap of sets

$A \cap B$ (A∩B) = the set of all elements that occur in both set A and set B

Relative Complement – the set of all elements that are in one set but not the other

$A - B$ (A / B) = the set of all elements that are in set A but are not in set B

$A=\{1,2,3,4,5\}$; $B=\{4,5,6,7\}$; $C=\{6,7,8\}$

$A \cup B = \{1,2,3,4,5,6,7\}$

$A \cap B = \{4,5\}$

$A \cap C = \emptyset$

$B \cup C = \{4,5,6,7,8\}$

$A \cup B \cup C = \{1,2,3,4,5,6,7,8\}$

$(A \cap B) \cup C = \{4,5,6,7,8\}$

$A \cup \emptyset = A$

$B \cap \emptyset = \emptyset$

$A / B = A - B = \{1,2,3\}$

$A - C = A$

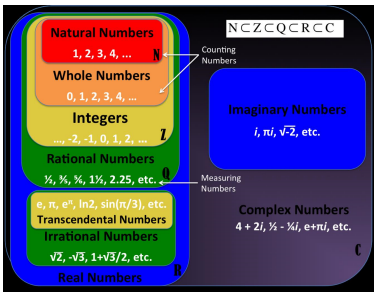
Print full name neatly — 15 August 2018
Intermediate Algebra, 2nd period — Quiz #2

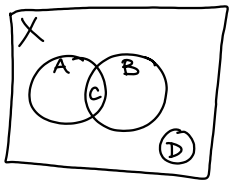
1. Give an example of a set written using the roster method. $\{1,2,3,4,5\}$

2. What are rational numbers? $\{a\}$ $\{1,2,\ldots\}$

#'s that can be written as fractions — terminating or repeating decimals

$\{\frac{p}{q} \mid p,q \in \mathbb{Z}\} = \mathbb{Q}$





$A \cap X = A$

$A \cup X = X$

$A \cap D = \emptyset$

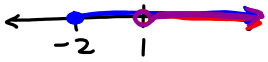
$B \setminus D = B$

$A \cap B = C$

1.1

77. $\{x \mid x > 1\} \cap \{x \mid x \geq -2\} = \{x \mid x > 1\}$

$= (1, \infty)$

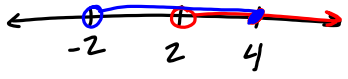


75. $\{x \mid x \leq 2\} \cap \{x \mid x \geq 0\} = [0,2]$

$= \{x \mid 0 \leq x \leq 2\}$

$= \{x \mid x \geq 0 \text{ and } x \leq 2\}$

113. $(2,\infty) \cup (-2,4] = (-2,\infty)$



$= \{x \mid x > -2\}$

~~$-2 < x < \infty$~~

Intermediate Algebra - 2nd period - Quiz #3
17 August 2018

1. Write this interval in set-builder notation:
   $(-3,5] = \{x \mid -3 < x \leq 5\}$

Given $A = \{-3,-1,1,3\}$ and $B = \{1,2,3,4\}$

2. Find $A \cup B$. $= \{-3,-1,1,2,3,4\}$
3. Find $A \cap B$. $= \{1,3\}$

An identity element is the number that we can apply to any other number that leaves it unchanged.

Additive Identity 0
the number that we can add to any other number that leaves it unchanged

$0 + x = x = x + 0$

An inverse element is the number that we can apply to an element that results in the identity element.

Additive Inverse of x $-x$
the number that we can add to an element that results in the identity element $x + (-x) = 0 = (-x) + x$

Multiplicative Identity 1

$1 \cdot x = x = x \cdot 1$

Multiplicative Inverse of x

$x \cdot \frac{1}{x} = 1 = \frac{1}{x} \cdot x$

↑ reciprocal

$-\frac{2}{3} \cdot \frac{1}{-\frac{2}{3}} = 1$

$\frac{1}{-\frac{2}{3}} = 1 \cdot -\frac{3}{2} = -\frac{3}{2}$

Properties of Addition and Multiplication

1. Commutativity

addition: $a+b=b+a$

multiplication: $ab=ba$

2. Associativity

addition: $(a+b)+c = a+(b+c)$

multiplication: $(ab)c = a(bc)$

3. Distributive Property of multiplication over addition

$a(b+c) = ab+ac$

* factoring a term from a polynomial applying the distributive property

1.4 Verbal Expressions and Variable Expressions

Translate into a variable expression and simplify: "The sum of half of a number and 6 less than twice that number."

$\frac{1}{2}x + (2x - 6)$ not necessary

or $\frac{x}{2}$ → $\frac{5}{2}x - 6$

$2\frac{1}{2} = \frac{5}{2}$

The sum of two numbers is 20. Using x to represent the smaller number, translate "the difference between five times the larger number and three less than the smaller number."

$x + y = 20$

larger #: $20 - x$

$5(20-x) - (x-3)$

$100 - 5x - x + 3 = 103 - 6x$

Order of Operations

P	E	M	D	A	S
Please	Excuse	My	Dear	Aunt	Sally
parentheses	exponentiation	multiplication	division	addition	subtraction

$5+2(3-4)-6(1*3)=$

Properties of Equality

1. Reflexivity

$x = x$

2. Symmetry

If $a=b$, then $b=a$

3. Transitivity

If $a=b$ and $b=c$, then $a=c$.

4. Substitution - if two things (numbers, expressions, etc.) are equal to each other, then they can be substituted for each other in any expression.

Intermediate Algebra - 2nd period - Quiz #4
20 August 2018

1. State the Commutative Property for Addition of real numbers.

$a+b = b+a$

2. What is the Multiplicative Identity?

1 $(a \cdot 1 = a)$

1.3

99. $-\frac{3}{4} \div \frac{5}{8} \cdot \left(-\frac{10}{11}\right)$

$+\frac{3}{4} \cdot \frac{8}{5} \cdot +\frac{10}{11} = \frac{12}{11}$

134.

$$\frac{1}{\left(\frac{1}{2}-\frac{1}{\left(\frac{1}{2}-\frac{1}{\left(1-\frac{1}{2}\right)}\right)}\right)}=\frac{1}{\frac{1}{2}-\frac{1}{\frac{1}{2}-\frac{1}{1/2}}}=$$

$1\div\frac{1}{2}=1\cdot\frac{2}{1}=2$

$$=\frac{1}{\frac{1}{2}-\frac{1}{\frac{1}{2}-2\cdot\frac{2}{2}}}=\frac{1}{\frac{1}{2}-\frac{1}{-\frac{3}{2}}}=$$

$$=\frac{1}{\frac{1}{2}-\left(-\frac{2}{3}\right)}=\frac{1}{\frac{1}{2}\cdot\frac{3}{3}+\frac{2}{3}\cdot\frac{2}{2}}=\frac{1}{\frac{3}{6}+\frac{4}{6}}$$

$$=\frac{1}{7/6}=\boxed{6/7}$$

127.

$$\frac{\left(\frac{(1-2\cdot3)}{(4(5-4))}\right)}{\left(\frac{(3-5\cdot2)}{(3\cdot5-1)}\right)}=\frac{\frac{-5}{4}}{\frac{-7}{14}}=\frac{-5}{4}\cdot\frac{-14}{7}$$

$$=\boxed{\frac{-5}{2}}$$

2.3 Value Mixture and Motion Problems $16\cdot\frac{\$}{16}=\$$

4. A coffee merchant combines coffee costing $5.50 per pound with coffee costing $3.00 per pound. How many pounds of each should be used to make 40 pounds of a blend costing $4.00 per pound?

Type of coffee	Weight	Cost per pound	Total cost
$5.50	x	5.5	$5.5x$
$3.00	$40-x$	3	$3(40-x)$
$4.00 blend	40	4	$4(40)$

$$5.5x+3(40-x)=4(40)$$

$$5.5x+120-3x=160$$

$$2.5x=40$$

$$x=\frac{40}{2.5}\cdot\frac{10}{10}=\frac{400}{25}=16$$

16 = weight of $5.50 coffee

24 = weight of $3 coffee

10. A silversmith combined pure silver that costs $5.20 an ounce with 50 ounces of a silver alloy that costs $2.80 an ounce. How many ounces of the pure silver were used to make an alloy of silver that costs $4.40 an ounce?

Type of metal	Weight (oz)	Cost per ounce	Total cost
Pure silver	x	5.2	$5.2x$
$2.80 alloy	50	2.8	$2.8(50)$
$4.40 alloy	$50+x$	4.4	$4.4(50+x)$

$$5.2x+2.8(50)=4.4(50+x)$$

$$9.2x+140=220+4.4x$$

$$0.8x=80$$

$$x=\frac{80}{0.8}=100$$ oz of pure silver

18. A chemist mixed 100 ml of an 8% saline solution with 60 ml of a 5% saline solution. Find the percent concentration of the resulting mixture.

solutions	% concentration	volume	amount of salt
8%	0.08	100 mL	$0.08(100)$
5%	0.05	60 mL	$0.05(60)$
mixture	x	160 mL	$160x$

$$0.08(100)+0.05(60)=160x$$