Radian Measure

The circumference of a circle of radius r is given by the equation:

$c = 2\pi r$

Therefore, the unit circle, which has radius 1, has circumference:

2π

The irrational number pi is approximately: $\pi \approx$

$\boxed{3.14}$15926535897932384626

433832795028841971...

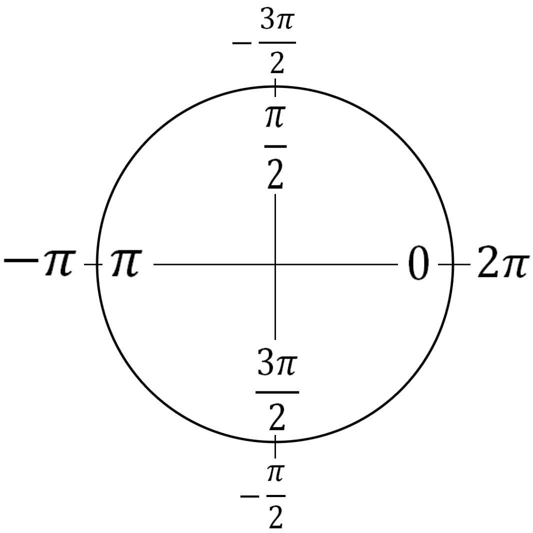
Therefore $2\pi \approx$

6.28

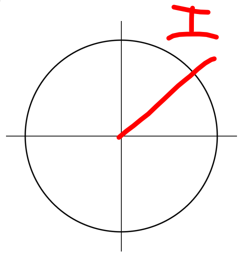
$4\pi \approx$

12.56

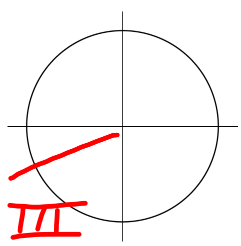
If we think about these numbers as corresponding to arc lengths around the unit circle, in which quadrant (or on which axis) do we end up?



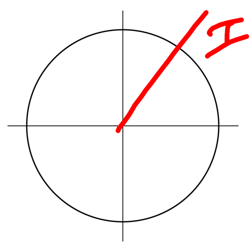
$\frac{\pi}{4}$



$\frac{7\pi}{6}$



$\frac{7\pi}{3} = \frac{6\pi}{3} + \frac{\pi}{3}$



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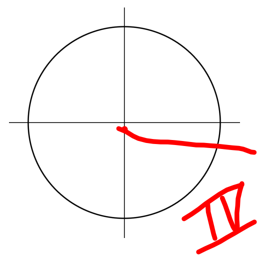

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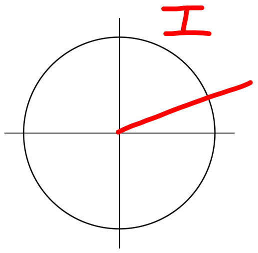
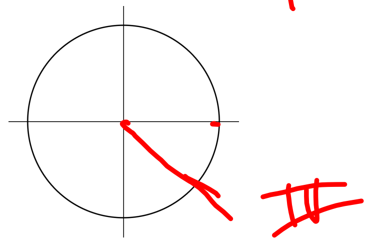
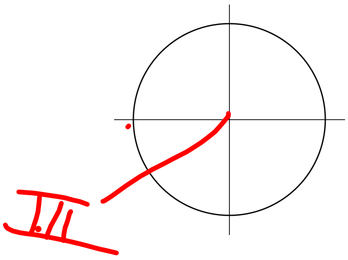


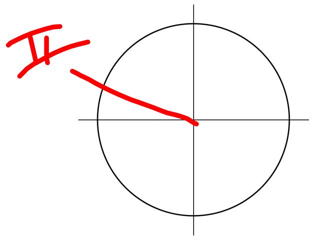
$-\frac{9\pi}{4} = -\frac{8\pi}{4} - \frac{\pi}{4}$



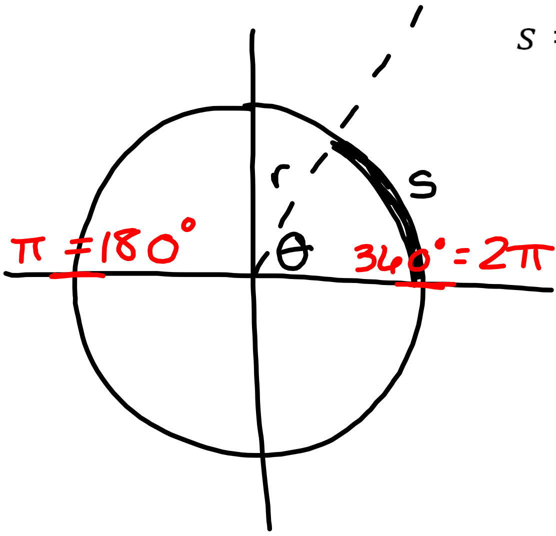
$\frac{19\pi}{6} = \frac{18\pi}{6} + \frac{\pi}{6}$



$\frac{6\pi}{7}$



What is a radian?



r = radius length

s = arc length

When $s = r$, we say that the corresponding angle θ which is subtended by arc s has measure 1 radian.

1 radian $\approx 57.3°$

$\pi = 180°$

$2\pi = 360°$

Note that θ is independent of the radius length and any unit of measurement. Therefore radians have no associated units, and any angle measure without a degree symbol is assumed to be in radians.

Converting between radians and degrees

$$\pi = 180° \quad \therefore \quad \frac{\pi}{180°} = 1 = \frac{180°}{\pi}$$

Convert 225° to radians.

$$\cancel{225}^{\cancel{45}\,5}° \cdot \frac{\pi}{\cancel{180}°_{\cancel{36}\,4}} = \boxed{\frac{5\pi}{4}}$$

Convert $\frac{5\pi}{6}$ to degrees.

$$\frac{5\cancel{\pi}}{\cancel{6}} \cdot \frac{\cancel{180}^{30}°}{\cancel{\pi}} = \boxed{150°}$$

Convert 120° to radians.

$$120\cancel{°} \cdot \frac{\pi}{180\cancel{°}} = \boxed{\frac{2\pi}{3}}$$

Convert $\frac{7\pi}{4}$ to degrees.

$$\frac{7\pi}{4} \cdot \frac{180°}{\pi} = \boxed{315°}$$

Two angles in radians are:

complementary if they sum to $\frac{\pi}{2}$. or $90°$

supplementary if they sum to π. or $180°$

coterminal if they differ by integer multiples of 2π. or $360°$

Find the complement and supplement of $\frac{5\pi}{12}$.

C: $\frac{\pi}{2} - \frac{5\pi}{12} = \frac{6\pi}{12} - \frac{5\pi}{12} = \boxed{\frac{\pi}{12}}$

S: $\pi - \frac{5\pi}{12} = \frac{12\pi}{12} - \frac{5\pi}{12} = \boxed{\frac{7\pi}{12}}$

Find one positive and one negative angle coterminal with $-\frac{3\pi}{4}$. $+2\pi$

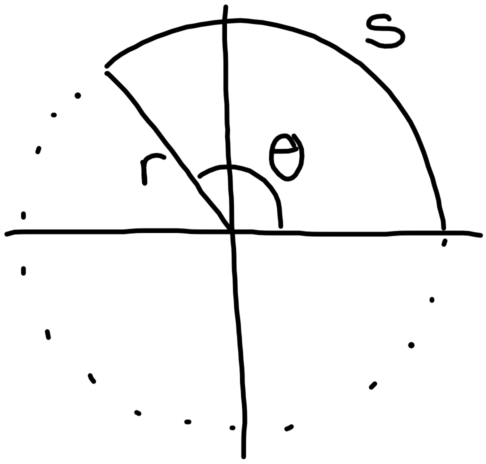
$+\frac{8\pi}{4}$

$\frac{-11\pi}{4}, \frac{5\pi}{4}$

complement of 1 :

$\frac{\pi}{2} - 1$

Arc Length & Angular Speed



Arc Length

r = radius or distance from the center of rotation $(in, cm, km, etc.)$

s = arc length or distance traveled along the circumference of a circle $(in, cm, km, etc.)$

θ = angle or amount of rotation $(deg, rad, revolutions, etc.)$

$$\boxed{s = r\theta}$$

1. $r = 5in$; $\theta = 45°$; $s = ?\,in$

$s = r\theta = 5in \cdot \cancel{45°}^{1} \cdot \dfrac{\pi}{\cancel{180°}_{4}} = \boxed{\dfrac{5\pi}{4}\, in}$

2. $s = 16yards$; $\theta = 5$; $r = ?\,yards$

$\dfrac{s}{\theta} = \dfrac{r\cancel{\theta}}{\cancel{\theta}}$ $\quad r = \dfrac{s}{\theta} = \dfrac{16\,yd}{5} = \boxed{\dfrac{16}{5}\,yd}$

3. Find the measure of a rotation in radians when a point 2 meters from the center of rotation travels 4 meters.

$\theta = ?\text{ rad}$; $r = 2m$; $s = 4m$

$\frac{s}{r} = \frac{\cancel{r}\theta}{\cancel{r}}$ $\quad \theta = \frac{s}{r} = \frac{4\cancel{m}}{2\cancel{m}} = \boxed{2}$

Linear Speed

$$v = \frac{s}{t}$$

Angular Speed

$$\omega = \frac{\theta}{t}$$

Arc Length

$$s = r\theta$$

Relating Linear & Angular Speed

$v = \frac{s}{t} = \frac{r\theta}{t} = r \cdot \frac{\theta}{t} = r\omega$

$\boxed{v = r\omega}$

$\boldsymbol{r}$ = radius or distance from the center of rotation $(in, cm, km, etc.)$

$\boldsymbol{s}$ = arc length or linear distance along the circumference of a circle $(in, cm, km, etc.)$

$\boldsymbol{\theta}$ = angle or amount of rotation $(deg, rad, revolutions, etc.)$

$\boldsymbol{t}$ = time $(sec, min, hours, years, etc.)$

$\boldsymbol{v} = \frac{\text{linear distance}}{\text{time}}$ = linear speed $\left(\frac{km}{s}, \frac{mi}{h}, etc.\right)$

$\boldsymbol{\omega} = \frac{\text{amount of rotation}}{\text{time}}$ = angular speed $\left(\frac{rev}{min}, \frac{deg}{s}, etc.\right)$

Handout Problems:

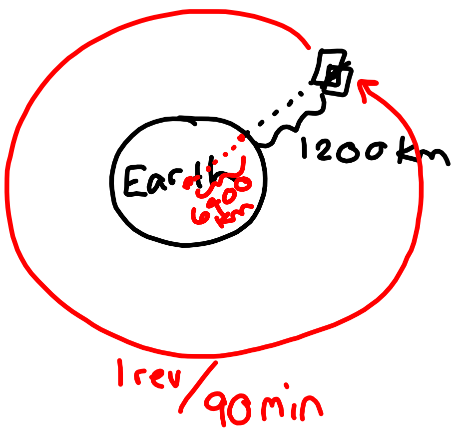
1. A wheel with a 15 inch diameter rotates at a rate of 6 radians per second. What is the linear speed of a point on its rim in feet per minute?

$r = \frac{15\text{in}}{2}$ $\omega = \frac{6\text{ rad}}{s}$ $V = ?\ \text{ft/min}$

$$V = r\omega = \frac{15\ \cancel{\text{in}}}{\cancel{2}} \cdot \frac{\overset{3}{\cancel{6}}\ \cancel{\text{rad}}}{\cancel{s}} \cdot \frac{1\text{ ft}}{\cancel{12}\ \cancel{\text{in}}} \cdot \frac{\overset{5}{\cancel{60}}\ \cancel{s}}{1\text{ min}}$$

$$= \boxed{225\ \text{ft/min}}$$

2. An earth satellite in circular orbit 1200 km high makes one complete revolution every 90 minutes. What is its linear speed in km/min, given that the earth's radius is 6400 km?



$r = 6400\text{km} + 1200\text{ km} = 7600\text{ km}$

$\omega = \frac{1\text{ rev}}{90\text{ min}}$

$V = ?\ \text{km/min}$

$$V = r\omega = 760\cancel{0}\text{km} \cdot \frac{1\ \cancel{\text{rev}}}{9\cancel{0}\text{ min}} \cdot \frac{2\pi}{1\ \cancel{\text{rev}}} = \boxed{\frac{1520\pi}{9}\ \text{km/min}}$$

Homework due this Friday:

Already assigned:
- 5.1 #1, 2, 7-18 all

New:
- **5.1 #31-48 all**
- **4 problems on handout**
- **5.1 #55-74 all**

Due next Wednesday, 11/13:
"Do you know enough Algebra..." take-home quiz