Chapter 6 - Trigonometric Identities and Equations

Reciprocal Identities

$\csc x = \frac{1}{\sin x}$, $\sin x = \frac{1}{\csc x}$

$\sec x = \frac{1}{\cos x}$, $\cos x = \frac{1}{\sec x}$

$\cot x = \frac{1}{\tan x}$, $\tan x = \frac{1}{\cot x}$

Cofunction Identities

$\sin\left(\frac{\pi}{2} - x\right) = \cos x$, $\cos\left(\frac{\pi}{2} - x\right) = \sin x$

$\tan\left(\frac{\pi}{2} - x\right) = \cot x$, $\cot\left(\frac{\pi}{2} - x\right) = \tan x$

$\csc\left(\frac{\pi}{2} - x\right) = \sec x$, $\sec\left(\frac{\pi}{2} - x\right) = \csc x$

Ratio Identities

$\tan \mathrm{x} = \frac{\sin \mathrm{x}}{\cos \mathrm{x}}$, $\cot x = \frac{\cos x}{\sin x}$

Odd-Even Identities

$\cos(-x) = \cos x$, $\sin(-x) = -\sin x$, $\tan(-x) = -\tan x$

$\sec(-x) = \sec x$, $\csc(-x) = -\csc x$, $\cot(-x) = -\cot x$

Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$

$1 + \cot^2 x = \csc^2 x$

$\tan^2 x + 1 = \sec^2 x$

Useful formulas from Algebra:

$$a^2 - b^2 = (a-b)(a+b)$$

$(a+b)^2 \neq a^2 + b^2$

$(a+b)(a+b)$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$a^3 - \cancel{a^2b} + \cancel{ab^2} + \cancel{a^2b} - \cancel{ab^2} + b^3$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$\cancel{a^3 + b^3 \neq (a+b)^3}$

6.2 - Sum and Difference Identities

$$\sin(a+b) = \sin a\cos b + \cos a\sin b$$

$$\sin(a-b) = \sin a\cos b - \cos a\sin b$$

$$\cos(a+b) = \cos a\cos b - \sin a\sin b$$

$$\cos(a-b) = \cos a\cos b + \sin a\sin b$$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a\tan b}$$

$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a\tan b}$$

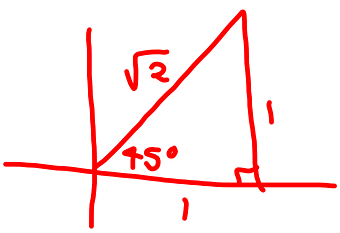
2. $\sin 375° = \sin(330° + 45°)$

$$= \sin 330° \cos 45° + \cos 330° \sin 45°$$

$$\cancel{\sin\left(-\tfrac{1}{2}\right)\cos\left(\tfrac{\sqrt{2}}{2}\right)}$$

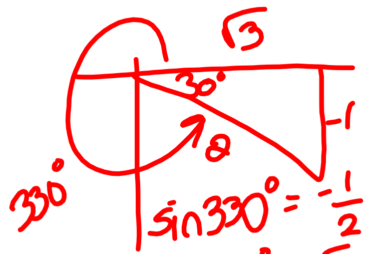
$$= \left(\frac{-1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{-\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$$



$\sin 45° = \cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$



$\sin 330° = -\frac{1}{2}$

$\cos 330° = \frac{\sqrt{3}}{2}$

$\cos\left(\frac{-\pi}{12}\right)$

10. $\cos\left(\frac{\pi}{4}-\frac{\pi}{3}\right)=$

$=\cos\frac{\pi}{4}\cos\frac{\pi}{3}+\sin\frac{\pi}{4}\sin\frac{\pi}{3}$

$=\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)+\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$=\boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$

14. $\underset{a}{\sin 167^\circ}\,\underset{b}{\cos 107^\circ}-\underset{a}{\cos 167^\circ}\,\underset{b}{\sin 107^\circ}$

$=\sin(167^\circ-107^\circ)$

$=\sin(60^\circ)$

$=\boxed{\frac{\sqrt{3}}{2}}$

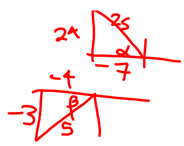
20. $\sin x \cos 3x + \cos x \sin 3x$

$= \sin(x+3x)$

$= \sin(4x)$

(34.) Given $\sin\alpha = \frac{24}{25}$, $\alpha \in Q\,\text{II}$

$\cos\beta = \frac{-4}{5}$, $\beta \in Q\,\text{III}$



Find $\sin(\alpha-\beta)$, $\cos(\alpha-\beta)$, $\tan(\alpha-\beta)$ & determine the quadrant in which $\alpha-\beta$ lies.

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= \left(\frac{24}{25}\right)\left(\frac{-4}{5}\right) - \left(\frac{-7}{25}\right)\left(\frac{-3}{5}\right)$

$= \frac{-96}{125} - \frac{21}{125} = \boxed{\frac{-117}{125}}$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$= \left(\frac{-7}{25}\right)\left(\frac{-4}{5}\right) + \left(\frac{24}{25}\right)\left(\frac{-3}{5}\right)$

$= \frac{28}{125} - \frac{72}{125} = \boxed{\frac{-44}{125}}$

$\tan(\alpha-\beta) = \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)} = \frac{\frac{-117}{125}}{\frac{-44}{125}} = \frac{-117}{125} \cdot \frac{125}{-44}$

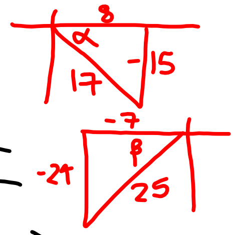
$= \boxed{\frac{117}{44}}$

$\alpha-\beta \in Q\,\text{III}$

II	I
(−, +)	(+, +)
III (−, −)	IV (+, −)

40. Given $\cos\alpha = \frac{8}{17}$, $\alpha \in Q\,IV$

$\sin\beta = \frac{-24}{25}$, $\beta \in Q\,III$



find $\sin(\alpha+\beta)$, $\cos(\alpha+\beta)$, $\tan(\alpha+\beta)$, & determine the quadrant in which $\alpha+\beta$ lies.

*Pythagorean triples that are useful to know:

3,4,5 ; 5,12,13 ; 7,24,25 ;

& 8,15,17

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \left(\frac{-15}{17}\right)\left(\frac{-7}{25}\right) + \left(\frac{8}{17}\right)\left(\frac{-24}{25}\right) = \boxed{\frac{-87}{425}}$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \left(\frac{8}{17}\right)\left(\frac{-7}{25}\right) - \left(\frac{-15}{17}\right)\left(\frac{-24}{25}\right) = \boxed{-\frac{416}{425}}$

$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \boxed{\frac{87}{416}}$

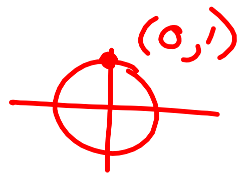
$\alpha+\beta \in Q\,\boxed{III}$

Cofunction Identities

The function of an angle is equal to the cofunction of its complement.

θ & $90° - \theta$ or θ & $\frac{\pi}{2} - \theta$

are complementary angles



$\cos\left(\frac{\pi}{2} - x\right) = \cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x$

$= 0 \cdot \cos x + 1 \cdot \sin x$

$= \boxed{\sin x}$

Double-Angle Identities

$\sin 2\theta = \sin(\theta + \theta)$

$= \sin\theta\cos\theta + \cos\theta\sin\theta$

$\boxed{\sin 2\theta = 2\sin\theta\cos\theta}$

$\sin(42x) = \sin[2(21x)] = 2\sin 21x \cos 21x$

$\sin(8x) = \sin 2[4x] = 2\sin 4x\cos 4x$

$= 2[\sin 2(2x)]\cos 4x$

$= 2[2\sin 2x\cos 2x]\cos 4x$

$= 2[2(2\sin x\cos x)\cos 2x]\cos 4x$

$\boxed{\sin 2\theta = 2\sin\theta\cos\theta}$

The sine of twice any angle is equal to two times the sine of that angle times the cosine of that angle.

$\sin 6\theta =$

$\sin 8\theta =$

$\sin 44\theta =$

$\sin 3\theta$

$= \sin(2\theta + \theta)$

even multiple ⇒ double ∠'s

odd multiple ⇒ sum

Homework #6/7:

- 6.1 #1-69 odd (proofs)
- 6.2 #1-41 odd due Wed
- 6.3 #1-24 all; 30-36 all; 49-93 odd

due after break

& **memorize your identities!!!**