

Double-Angle Identities

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sin^2 \theta = 1 - \cos^2 \theta$$

$$\cos^2 \theta = 1 - \sin^2 \theta$$

$$\cos 2\theta = \cos(\theta + \theta) = \cos \theta \cos \theta - \sin \theta \sin \theta$$

$$\cos 2\theta = \underline{\cos^2 \theta} - \underline{\sin^2 \theta}$$

$$= \underline{1 - \sin^2 \theta} - \sin^2 \theta$$

$$\cos 2\theta = 1 - 2 \sin^2 \theta$$

$$= \cos^2 \theta - (1 - \cos^2 \theta)$$

$$\cos 2\theta = 2 \cos^2 \theta - 1$$

$$\tan 2\theta = \tan(\theta + \theta)$$

$$= \frac{\tan \theta + \tan \theta}{1 - \tan \theta \tan \theta}$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Double-Angle Identities

$$\sin 2x = 2 \sin x \cos x$$

$$\begin{aligned}\cos 2x &= \cos^2 x - \sin^2 x \\ &= 2 \cos^2 x - 1 \\ &= 1 - 2 \sin^2 x\end{aligned}$$

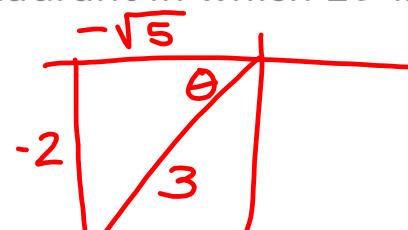
$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

Given $\sin \theta = -\frac{2}{3}$, $\theta \in QIII$,

Find $\sin 2\theta$, $\cos 2\theta$, $\tan 2\theta$, and the quadrant in which 2θ lies.

$$\begin{aligned}\sin 2\theta &= 2 \sin \theta \cos \theta \\ &= 2 \left(-\frac{2}{3}\right) \left(\frac{-\sqrt{5}}{3}\right)\end{aligned}$$

$$\boxed{\sin 2\theta = \frac{4\sqrt{5}}{9}}$$



$$\boxed{\cos 2\theta = \frac{1}{9}}$$

$$\begin{aligned}\cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\ &= \left(\frac{-\sqrt{5}}{3}\right)^2 - \left(\frac{-2}{3}\right)^2 = \frac{5}{9} - \frac{4}{9} = \frac{1}{9}\end{aligned}$$

$$\boxed{\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = 4\sqrt{5}}$$

$$\boxed{2\theta \in QI}$$

Half-Angle Identities

$$\sin \frac{x}{2} = ?$$

$$\cos 2\theta = 1 - 2\sin^2 \theta$$

$$\text{Let } \theta = \frac{x}{2}$$

$$\cos(2 \cdot \frac{x}{2}) = 1 - 2\sin^2 \frac{x}{2}$$

$$\cos x = 1 - 2\left(\sin \frac{x}{2}\right)^2$$

$$2\left(\sin \frac{x}{2}\right)^2 = 1 - \cos x$$

$$\left(\sin \frac{x}{2}\right)^2 = \frac{1 - \cos x}{2}$$

$$\boxed{\sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}}}$$

$$\cos \frac{x}{2} = ?$$

$$\cos 2\theta = 2\cos^2 \theta - 1$$

$$\text{Let } \theta = \frac{x}{2}$$

$$\cos 2 \cdot \frac{x}{2} = 2\cos^2 \frac{x}{2} - 1$$

$$\cos x = 2\cos^2 \frac{x}{2} - 1$$

$$\cos x + 1 = 2\cos^2 \frac{x}{2}$$

$$\frac{1 + \cos x}{2} = \cos^2 \frac{x}{2}$$

$$\boxed{\cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}}$$

Half-Angle Identities

$$\sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}}, \quad \cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}$$

$$\tan \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

$$= \frac{\sin x}{1 + \cos x}$$

$$\boxed{\tan \frac{x}{2} = \frac{1 - \cos x}{\sin x}}$$

$$\begin{aligned} \tan \frac{7\pi}{12} &= \tan \frac{\frac{7\pi}{6}}{2} \\ &= \frac{1 - \cos \frac{7\pi}{6}}{\sin \frac{7\pi}{6}} \\ &= \frac{1 - \left(-\frac{\sqrt{3}}{2}\right)}{-\frac{1}{2}} \\ &= \frac{\frac{2}{2} + \frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \frac{2 + \sqrt{3}}{2} \cdot \frac{-2}{1} \\ &= \boxed{-2 - \sqrt{3}} \end{aligned}$$

$$\begin{aligned} \frac{7\pi}{12} &= \frac{x}{2} \\ \frac{7\pi}{6} &= x \\ \tan \frac{x}{2} &= \frac{1 - \cos x}{\sin x} \\ \text{Graph: } &\quad \begin{array}{c} \text{---} \\ | \quad | \\ -1 \quad 2 \\ | \quad | \\ \text{---} \end{array} \quad \begin{array}{c} \text{---} \\ | \quad | \\ -\sqrt{3} \quad 1 \\ | \quad | \\ \text{---} \end{array} \quad \begin{array}{c} \text{---} \\ | \quad | \\ 30^\circ \quad 90^\circ \\ | \quad | \\ \text{---} \end{array} \end{aligned}$$

6.3 Evaluate using the half-angle identity.

14. $\sin 112.5^\circ = \sin \frac{225^\circ}{2}$

$$\begin{aligned} &= + \sqrt{\frac{1 - \cos 225^\circ}{2}} \\ \begin{matrix} b/c \\ 112.5^\circ \\ \in QII \end{matrix} &= \sqrt{\frac{1 - \left(-\frac{\sqrt{2}}{2}\right)}{2}} \\ \begin{matrix} \sqrt{2} \\ \sqrt{2} \\ \sqrt{2} \end{matrix} &= \sqrt{\frac{\frac{2}{2} + \frac{\sqrt{2}}{2}}{2}} \\ &= \sqrt{\frac{2 + \sqrt{2}}{2} \cdot \frac{1}{2}} = \sqrt{\frac{2 + \sqrt{2}}{4}} \\ &= \boxed{\frac{\sqrt{2 + \sqrt{2}}}{2}} \end{aligned}$$

$$112.5^\circ = \frac{225^\circ}{2}$$

$$\sin \frac{x}{2} = + \sqrt{\frac{1 - \cos x}{2}}$$

$$\begin{aligned} \text{Graph: } &\quad \begin{array}{c} \text{---} \\ | \quad | \\ -1 \quad \sqrt{2} \\ | \quad | \\ \text{---} \end{array} \quad \begin{array}{c} \text{---} \\ | \quad | \\ 225^\circ \quad 270^\circ \\ | \quad | \\ \text{---} \end{array} \quad \begin{array}{c} \text{---} \\ | \quad | \\ \cos 225^\circ = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2} \\ | \quad | \\ \text{---} \end{array} \end{aligned}$$

6.3 Prove/Verify the identity.

$$50. \cos 8x = \cos^2 4x - \sin^2 4x$$

$$\text{LHS} = \cos[2(4x)] = \cos^2 4x - \sin^2 4x = \text{RHS} \checkmark$$

$$52. \frac{\cos 2x}{\sin^2 x} = \cot^2 x - 1$$

$$\begin{aligned}\text{LHS} &= \frac{\cos^2 x - \sin^2 x}{\sin^2 x} \\ &= \frac{\cos^2 x}{\sin^2 x} - \frac{\sin^2 x}{\sin^2 x} \\ &= \cot^2 x - 1 \\ &= \text{RHS} \checkmark \\ &\text{Yay!}\end{aligned}$$

$$\begin{aligned}\cos 2x &= \cos^2 x - \sin^2 x \\ &= 2\cos^2 x - 1 \\ &= 1 - \sin^2 x\end{aligned}$$

~~$\frac{x-2}{2}$~~ bad!

$$\begin{aligned}\frac{2x-4}{4} &\\ \cancel{2(x-2)}_{\cancel{2}} &\text{OK!}\end{aligned}$$

$$54. \frac{1}{1-\cos 2x} = \frac{1}{2} \csc^2 x$$

$$\text{LHS} = \frac{1}{1-(1-2\sin^2 x)} = \frac{1}{2\sin^2 x} = \frac{1}{2} \csc^2 x = \text{RHS}$$

$$56. \frac{\cos^2 x - \sin^2 x}{2\sin x \cos x} = \cot 2x$$

$$\text{LHS} = \frac{\cos 2x}{\sin 2x} = \cot 2x = \text{RHS}$$

Yay!

$$60. \sin 2x - \cot x \cos 2x = -\cot x \cos 2x$$

$$\text{LHS} = 2\sin x \cos x - \frac{\cos x}{\sin x}$$

$$= 2\sin x \cos x \cdot \frac{\sin x}{\sin x} - \frac{\cos x}{\sin x}$$

$$= \frac{2\sin^2 x \cos x - \cos x}{\sin x}$$

$$= -\frac{\cos x(1 - 2\sin^2 x)}{\sin x} = -\frac{\cos x}{\sin x} \cdot \frac{1 - 2\sin^2 x}{1}$$

$$= -\cot x \cos 2x = \text{RHS} \checkmark$$

Homework #6:

- 6.1 #1-69 odd (proofs) due
after
break
- 6.2 #1-41 odd ~~due Wed~~ due
after
break
- 6.3 #1-24 all; 30-36 all; 49-93 odd

& memorize your identities!!!

Know Your Identities
Quiz: Wed?