Double-Angle Identities

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos(\theta+\theta) = \cos\theta\cos\theta - \sin\theta\sin\theta$

$\boxed{\cos 2\theta = \cos^2\theta - \sin^2\theta}$

$= 1 - \sin^2\theta - \sin^2\theta$

$\boxed{\cos 2\theta = 1 - 2\sin^2\theta}$

$= \cos^2\theta - (1 - \cos^2\theta)$

$\boxed{\cos 2\theta = 2\cos^2\theta - 1}$

$\tan 2\theta = \tan(\theta+\theta)$

$= \dfrac{\tan\theta + \tan\theta}{1 - \tan\theta\tan\theta}$

$\boxed{\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}}$

Double-Angle Identities

$$\sin 2x = 2\sin x\cos x$$

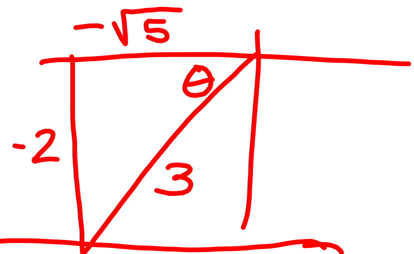
$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 2\cos^2 x - 1$$
$$= 1 - 2\sin^2 x$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

Given $\sin\theta = -\frac{2}{3}$, $\theta \in QIII$,

Find $\sin 2\theta$, $\cos 2\theta$, $\tan 2\theta$, and the quadrant in which 2θ lies.

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$= 2\left(\frac{-2}{3}\right)\left(\frac{-\sqrt{5}}{3}\right)$$



$$\boxed{\sin 2\theta = \frac{4\sqrt{5}}{9}}$$

$$\boxed{\cos 2\theta = \frac{1}{9}}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= \left(\frac{-\sqrt{5}}{3}\right)^2 - \left(-\frac{2}{3}\right)^2 = \frac{5}{9} - \frac{4}{9} = \frac{1}{9}$$

$$\boxed{\tan 2\theta} = \frac{\sin 2\theta}{\cos 2\theta} = \boxed{4\sqrt{5}}$$

$$\boxed{2\theta \in QI}$$

Half-Angle Identities

$\sin\frac{x}{2} = ?$

$\cos 2\theta = 1 - 2\sin^2\theta$

Let $\theta = \frac{x}{2}$.

$\cos\left(2\cdot\frac{x}{2}\right) = 1 - 2\sin^2\frac{x}{2}$

$\cos x = 1 - 2\left(\sin\frac{x}{2}\right)^2$

$2\left(\sin\frac{x}{2}\right)^2 = 1 - \cos x$

$\left(\sin\frac{x}{2}\right)^2 = \frac{1-\cos x}{2}$

$\boxed{\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}}$

$\cos\frac{x}{2} = ?$

$\cos 2\theta = 2\cos^2\theta - 1$

Let $\theta = \frac{x}{2}$

$\cos 2\cdot\frac{x}{2} = 2\cos^2\frac{x}{2} - 1$

$\cos x = 2\cos^2\frac{x}{2} - 1$

$\cos x + 1 = 2\cos^2\frac{x}{2}$

$\frac{1+\cos x}{2} = \cos^2\frac{x}{2}$

$\boxed{\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}}$

Half-Angle Identities

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}} \quad , \quad \cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$$

$$\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$$

$$= \frac{\sin x}{1+\cos x}$$

$$\boxed{\tan\frac{x}{2} = \frac{1-\cos x}{\sin x}}$$

$$\tan\frac{7\pi}{12} = \tan\frac{\frac{7\pi}{6}}{2}$$

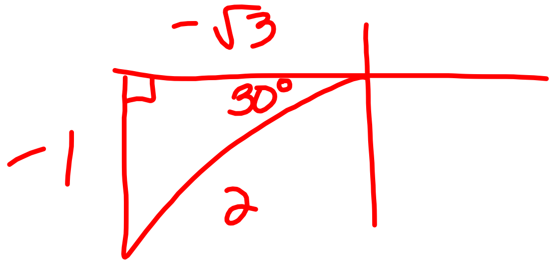
$$\frac{7\pi}{12} = \frac{x}{2}$$

$$\frac{7\pi}{6} = x$$

$$= \frac{1-\cos\frac{7\pi}{6}}{\sin\frac{7\pi}{6}}$$

$$\tan\frac{x}{2} = \frac{1-\cos x}{\sin x}$$

$$= \frac{1-\left(\frac{-\sqrt{3}}{2}\right)}{-\frac{1}{2}}$$



$$= \frac{\frac{2}{2}+\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \frac{2+\sqrt{3}}{2}\cdot\frac{-2}{1}$$

$$= \boxed{-2-\sqrt{3}}$$

6.3 Evaluate using the half-angle identity.

14. sin 112.5° $= \sin\frac{225°}{2}$

$$112.5° = \frac{225°}{2}$$

$$= +\sqrt{\frac{1-\cos 225°}{2}}$$

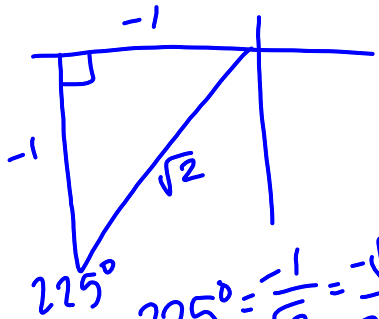
b/c $112.5° \in$ QII

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$$

$$= \sqrt{\frac{1-\left(\frac{-\sqrt{2}}{2}\right)}{2}}$$

$$\sqrt{\frac{M}{N}} = \frac{\sqrt{M}}{\sqrt{N}}$$

$$= \sqrt{\frac{\frac{2}{2}+\frac{\sqrt{2}}{2}}{\frac{2}{1}}}$$



$$= \sqrt{\frac{2+\sqrt{2}}{2}\cdot\frac{1}{2}} = \sqrt{\frac{2+\sqrt{2}}{4}}$$

$$\cos 225° = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$$

$$= \boxed{\frac{\sqrt{2+\sqrt{2}}}{2}}$$

6.3 Prove/Verify the identity.

50. $\cos 8x = \cos^2 4x - \sin^2 4x$

$LHS = \cos[2(4x)] = \cos^2 4x - \sin^2 4x = RHS$ ✓

52. $\dfrac{\cos 2x}{\sin^2 x} = \cot^2 x - 1$

$\boxed{\begin{aligned}\cos 2x &= \cos^2 x - \sin^2 x\\ &= 2\cos^2 x - 1\\ &= 1 - \sin^2 x\end{aligned}}$

$LHS = \dfrac{\cos^2 x - \sin^2 x}{\sin^2 x}$

$= \dfrac{\cos^2 x}{\sin^2 x} - \dfrac{\sin^2 x}{\sin^2 x}$

$= \cot^2 x - 1$

$= RHS$ ✓

Yay!

$\cancel{\dfrac{x-\cancel{2}}{\cancel{2}}}$ bad!

$\dfrac{2x-4}{4}$

$\dfrac{\cancel{2}(x-2)}{\cancel{2}\cdot 2}$ ok!

54. $\dfrac{1}{1-\cos 2x} = \dfrac{1}{2}\csc^2 x$

$LHS = \dfrac{1}{1-(1-2\sin^2 x)} = \dfrac{1}{2\sin^2 x} = \dfrac{1}{2}\csc^2 x = RHS$

56. $\dfrac{\cos^2 x - \sin^2 x}{2\sin x\cos x} = \cot 2x$

$LHS = \dfrac{\cos 2x}{\sin 2x} = \cot 2x = RHS$

Yay!

60. $\sin 2x - \cot x = -\cot x \cos 2x$

$$LHS = 2\sin x\cos x - \frac{\cos x}{\sin x}$$

$$= 2\sin x\cos x \cdot \frac{\sin x}{\sin x} - \frac{\cos x}{\sin x}$$

$$= \frac{2\sin^2 x\cos x - \cos x}{\sin x}$$

$$= \frac{-\cos x(1-2\sin^2 x)}{\sin x} = -\frac{\cos x}{\sin x} \cdot \frac{1-2\sin^2 x}{1}$$

$$= -\cot x\cos 2x = RHS \checkmark$$

Homework #6:

- 6.1 #1-69 odd (proofs)
- 6.2 #1-41 odd ~~⟵⟶~~ due Wed
- 6.3 #1-24 all; 30-36 all; 49-93 odd

due after break

& **memorize your identities!!!**

Know Your Identities

Quiz: Wed?