Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$

$1 + \cot^2 x = \csc^2 x$

$\tan^2 x + 1 = \sec^2 x$

Sum and Difference Identities

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\sin(a-b) = \sin a \cos b - \cos a \sin b$

$\cos(a+b) = \cos a \cos b - \sin a \sin b$

$\cos(a-b) = \cos a \cos b + \sin a \sin b$

$\tan(a+b) = \dfrac{\tan a + \tan b}{1 - \tan a \tan b}$

$\tan(a-b) = \dfrac{\tan a - \tan b}{1 + \tan a \tan b}$

Double-Angle Identities

$\sin 2x = 2 \sin x \cos x$

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$\tan 2x = \dfrac{2 \tan x}{1 - \tan^2 x}$

Half-Angle Identities

$\sin \frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$, $\cos \frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$

$$\tan \frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}} = \frac{\sin x}{1+\cos x}$$

$$\boxed{\tan \frac{x}{2} = \frac{1-\cos x}{\sin x}}$$

$$\frac{(3-\sqrt{3})}{3+\sqrt{3}} \cdot \frac{(3-\sqrt{3})}{3-\sqrt{3}} = \frac{9-6\sqrt{3}+3}{9-3}$$

$$= \frac{12-6\sqrt{3}}{6} = \frac{12}{6} - \frac{6\sqrt{3}}{6}$$

$$= 2-\sqrt{3}$$

62. $\sin 4x = 4\sin x\cos^3 x - 4\cos x\sin^3 x$

$LHS = \sin 2(2x) = 2\sin 2x\cos 2x$

$= 2(2\sin x\cos x)(\cos^2 x - \sin^2 x)$

$= 4\sin x\cos^3 x - 4\sin^3 x\cos x$

$= RHS$ ✓

64. $2\cos^4 x - \cos^2 x - 2\sin^2 x\cos^2 x + \sin^2 x = \cos^2 2x$

$RHS = (\cos 2x)^2 = (\cos 2x)(\cos 2x)$

$= (\cos^2 x - \sin^2 x)(2\cos^2 x - 1)$

$= 2\cos^4 x - \cos^2 x - 2\sin^2 x\cos^2 x + \sin^2 x$

$= LHS$ ✓

66. $\sin 4x = 4\sin x\cos x - 8\cos x\sin^3 x$

$LHS = \sin 2(2x) = 2\sin 2x\cos 2x =$

$= \underbrace{2(2\sin x\cos x)}_{4\sin x\cos x}\underbrace{(1-2\sin^2 x)} =$

$= 4\sin x\cos x - 8\sin^3 x\cos x =$

$= RHS$

68. $\sin 3x + \sin x = 4\sin x - 4\sin^3 x$

$\sin^2 x + \cos^2 x = 1$
$\cos^2 x = 1 - \sin^2 x$

$LHS = \sin(2x + x) + \sin x$

$= \underline{\sin 2x}\cos x + \underline{\cos 2x}\sin x + \sin x$

$= \underline{2\sin x\cos x} \cdot \cos x + (\underline{1-2\sin^2 x})\sin x + \sin x$

$= 2\sin x\underline{\cos^2 x} + \sin x - 2\sin^3 x + \sin x$

$= 2\sin x - 2\sin^3 x + 2\sin x(\underline{1-\sin^2 x})$

$= 2\sin x - 2\sin^3 x + 2\sin x - 2\sin^3 x$

$= 4\sin x - 4\sin^3 x$

$= RHS.$

72. $\cos^2\frac{x}{2} = \frac{\sec x + 1}{2\sec x}$

$LHS = \left(\cos\frac{x}{2}\right)^2 = \left(\pm\sqrt{\frac{1+\cos x}{2}}\right)^2 = \frac{1+\cos x}{2}$

$= \frac{1+\cos x}{2} \cdot \frac{\frac{1}{\cos x}}{\frac{1}{\cos x}} = \frac{\frac{1}{\cos x}+1}{\frac{2}{\cos x}} =$

$= \frac{\sec x + 1}{2\sec x} = RHS$

76. $\cos^2\frac{x}{2} - \sin^2\frac{x}{2} = \cos x$

$LHS = \left(\pm\sqrt{\frac{1+\cos x}{2}}\right)^2 - \left(\pm\sqrt{\frac{1-\cos x}{2}}\right)^2$

$= \frac{1+\cos x}{2} - \frac{1-\cos x}{2}$

$= \frac{2\cos x}{2} = \cos x = RHS$

86. $$\frac{\cos 2x}{\sin^2 x} = \csc^2 x - 2$$

$$LHS = \frac{1-2\sin^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} - \frac{2\sin^2 x}{\sin^2 x}$$

$$= \csc^2 x - 2 = RHS$$

$$\frac{\cos 2x}{1} \cdot \frac{1}{\sin^2 x} = (\cos 2x)\csc^2 x$$

$$= (1-2\sin^2 x)\csc^2 x$$

88. $$\frac{2\cos 2x}{\sin 2x} = \cot x - \tan x$$

$$LHS = \frac{\cancel{2}(\cos^2 x - \sin^2 x)}{\cancel{2}\sin x\cos x} = \frac{\cos^{\cancel{2}} x}{\sin x\cancel{\cos x}} - \frac{\sin^{\cancel{2}} x}{\cancel{\sin x}\cos x}$$

$\cos x \cancel{\cos x}$

$$= \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} = \cot x - \tan x = RHS$$

HW #6 (due Wed, 09/17)
- 6.2 #1-41 odd sum, difference, and cofunction identities

Quiz #5 - Wed, 09/17
- know statements of all identities
- apply sum, difference, double- & half-angles
- review questions (anything from test 1 or 2)

HW #7 (due Fri, 09/26)
- 6.1#1-69 odd proofs
- 6.3 #1-24 all double- and half-angle identities (application & proof)
  #30-36 all
  #49-93 odd
- 6.5 #1-24 all inverse functions