I. (1 point each) Evaluate the following trigonometric and inverse trigonometric expressions:

1. $\sin^{-1}\left(-\frac{1}{2}\right) =$ $-\pi/6$

2. $\tan\frac{7\pi}{4} =$ -1

3. $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) =$ $\frac{2\pi}{3}$

4. $\sec\frac{4\pi}{3} =$ -2

5. $\csc^{-1}(-\sqrt{2}) =$ $-\frac{\pi}{4}$

II. (5 points each) Verify (prove) the following trigonometric identities:

6. $\sin(a-b) - \sin(a+b) = -2\cos a \sin b$

$LHS = (\sin a\cos b - \cos a\sin b) - (\sin a\cos b + \cos a\sin b)$

$= -2\cos a\sin b$

$= RHS$

7. $\frac{2\cos 2x}{\sin 2x} = \cot x - \tan x$

$LHS = \frac{\cancel{2}(\cos^2 x - \sin^2 x)}{\cancel{2}\sin x\cos x} = \frac{\cos^{\cancel{2}} x}{\sin x\cancel{\cos x}} - \frac{\sin^{\cancel{2}} x}{\cancel{\sin x}\cos x}$

$= \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} = \cot x - \tan x = RHS$ ✓

82. $\cos 3x + \cos x = 0$

Solve for $x \in [0, 2\pi)$

$\cos(2x + x) + \cos x = 0$

$\cos 2x \cos x - \sin 2x \sin x + \cos x = 0$

$(\cos^2 x - \sin^2 x)\cos x - (2\sin x \cos x)\sin x + \cos x = 0$

$\cos^3 x - \sin^2 x \cos x - 2\sin^2 x \cos x + \cos x = 0$

$\cos^3 x - 3\sin^2 x \cos x + \cos x = 0$

$\cos^3 x - 3(1 - \cos^2 x)\cos x + \cos x = 0$

$\cos^3 x - 3\cos x + 3\cos^3 x + \cos x = 0$

$4\cos^3 x - 2\cos x = 0$

$2\cos x(2\cos^2 x - 1) = 0$

$2\cos x = 0$

$\cos x = 0$

$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}}$

$2\cos^2 x - 1 = 0$

$\cos^2 x = \frac{1}{2}$

$\cos x = \pm\frac{1}{\sqrt{2}}$

$\boxed{x = \pi/4, 3\pi/4, \frac{5\pi}{4}, \frac{7\pi}{4}}$

$x \in [0, 2\pi)$

84. $2\sin x\cos x - 2\sqrt{2}\sin x - \sqrt{3}\cos x + \sqrt{6} = 0$

$2\sin x(\cos x - \sqrt{2}) - \sqrt{3}(\cos x - \sqrt{2}) = 0$

$(\cos x - \sqrt{2})(2\sin x - \sqrt{3}) = 0$

$\cos x - \sqrt{2} = 0$, $2\sin x - \sqrt{3} = 0$

$\cancel{\cos x = \sqrt{2}}$

$\sin x = \frac{\sqrt{3}}{2}$

$\boxed{x = \pi/3, 2\pi/3}$

76. $\tan\frac{x}{2} = 1 - \cos x$

$x \in [0, 2\pi)$

$\tan\frac{x}{2} = \frac{1-\cos x}{\sin x} = \boxed{\frac{\sin x}{1+\cos x}}$

$\frac{\sin x}{1+\cos x} = 1 - \cos x$

$\frac{\sin x}{\cancel{1+\cos x}} \cdot (\cancel{1+\cos x}) = (1-\cos x)(1+\cos x)$

$\sin x = 1 - \cos^2 x$

$\sin x = \sin^2 x$

$0 = \sin^2 x - \sin x$

$0 = \sin x(\sin x - 1) \rightarrow \sin x = 0, \ \sin x = 1$

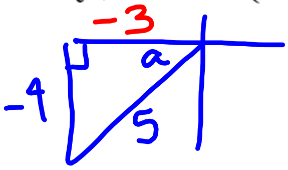
$x = \boxed{0, \pi, \frac{\pi}{2}}$

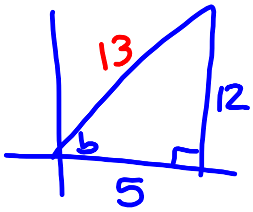
1. Given that $\sin a = -\frac{4}{5}$, $a \in QIII$, and $\tan b = \frac{12}{5}$, $b \in QI$, find $\sin(a-b)$.

$\sin(a-b) = \sin a\cos b - \cos a\sin b$

$= \left(\frac{-4}{5}\right)\left(\frac{5}{13}\right) - \left(\frac{-3}{5}\right)\left(\frac{12}{13}\right)$

$= \frac{-20}{65} + \frac{36}{65} = \boxed{\frac{16}{65}}$



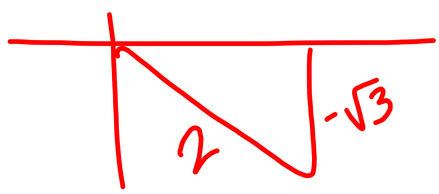


2. Simplify and express as a single trigonometric function $\frac{\csc^2 x - 2}{\csc^2 x}$.

$\frac{\csc^2 x}{\csc^2 x} - \frac{2}{\csc^2 x} = 1 - 2\sin^2 x = \boxed{\cos 2x}$

3. Evaluate $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$. Give the answer in radians.

$\boxed{\frac{-\pi}{3}}$

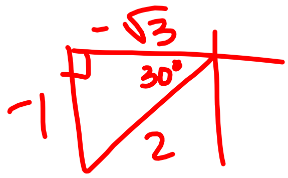


1. Use the half-angle identity to evaluate $\tan\frac{7\pi}{12}$ exactly.

$$\tan\frac{x}{2} = \frac{1-\cos x}{\sin x}$$

$$\frac{7\pi}{12} = \frac{x}{2}$$

$$\tan\frac{7\pi/6}{2} = \frac{1-\cos\frac{7\pi}{6}}{\sin\frac{7\pi}{6}} =$$



$$= \frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{\frac{-1}{2}} = \left(1+\frac{\sqrt{3}}{2}\right)\left(-\frac{2}{1}\right) = \boxed{-2-\sqrt{3}}$$

2. Find the exact value of $\cos 212° \cos 122° + \sin 212° \sin 122°$.

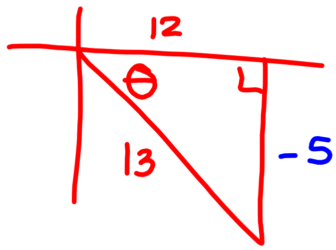
$$\cos(212° - 122°)$$

$$= \cos 90° = \boxed{0}$$

3. Find $\sin 2\theta, \cos 2\theta, and \tan 2\theta$ given that $\cos\theta = \frac{12}{13}$ and θ is in Quadrant IV.

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$= 2\left(\frac{-5}{13}\right)\left(\frac{12}{13}\right) = \boxed{\frac{-120}{169} = \sin 2\theta}$$



$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$= \left(\frac{12}{13}\right)^2 - \left(\frac{-5}{13}\right)^2$$

$$= \frac{144}{169} - \frac{25}{169} = \boxed{\frac{119}{169} = \cos 2\theta}$$

$$\boxed{2\theta \in Q\,\mathrm{IV}}$$

$$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{-120/169}{119/169} = \boxed{\frac{-120}{119} = \tan 2\theta}$$

Homework for Test #3:

Homework #6 (submitted Wed. 09/17)
- 6.2 #1-41 odd sum, difference, and cofunction identities

Homework #7 (submitted 09/26)
- 6.1 #1-69 odd proofs
- 6.3 #1-24 all double- and half-angle identities (application & proof)
  #30-36 all #49-93 odd
- 6.5 #1-24 all inverse functions #25-55 odd inverse functions

Homework #8 (due Friday 10/03)
- 6.6 #1-21 odd finding solutions between 0 and 2pi
- 6.6 #61-69 odd finding all possible solutions (+2pi*k)
- 6.6 #71-83 odd;
- Examples #3,4,7,8 from solving equations handout
- Test 3 Practice Problems handout

Test #3 - Friday 10/3