3. $\frac{1}{2}\csc^2\frac{x}{2} = \csc^2 x + \cot x \csc x$

Left-hand side $= \frac{1}{2}\left[\csc\left(\frac{x}{2}\right)\right]^2 = \frac{1}{2}\left[\frac{1}{\sin\left(\frac{x}{2}\right)}\right]^2 = \frac{1}{2}\left[\frac{1}{\pm\sqrt{\frac{1-\cos x}{2}}}\right]^2 =$

$$= \frac{1}{2}\left(\frac{1}{\frac{1-\cos x}{2}}\right) = \frac{1}{2}\cdot\frac{2}{1-\cos x} = \frac{1}{1-\cos x} =$$

$$= \frac{1}{1-\cos x}\cdot\frac{1+\cos x}{1+\cos x} = \frac{1+\cos x}{1-\cos^2 x} = \frac{1+\cos x}{\sin^2 x} =$$

$$= \frac{1}{\sin^2 x} + \frac{\cos x}{\sin^2 x} = \csc^2 x + \frac{\cos x}{\sin x \sin x} =$$

$$= \csc^2 x + \frac{\cos x}{\sin x}\cdot\frac{1}{\sin x} = \csc^2 x + \cot x \csc x =$$

=Right-hand side

4. $\sec 2x = \frac{\sec^2 x}{2 - \sec^2 x}$

Left-hand side $= \frac{1}{\cos 2x} = \frac{1}{2\cos^2 x - 1} = \frac{1}{2(\cos x)^2 - 1} =$

$$= \frac{1}{2\left(\frac{1}{\sec x}\right)^2 - 1} = \frac{1}{\frac{2}{\sec^2 x} - 1\cdot\frac{\sec^2 x}{\sec^2 x}} =$$

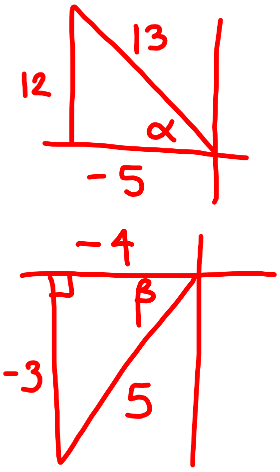
$$= \frac{1}{\frac{2-\sec^2 x}{\sec^2 x}} = 1\cdot\frac{\sec^2 x}{2-\sec^2 x} = \text{Right-hand side} \ \blacksquare$$

10. $\cos 3x = 4\cos^3 x - 3\cos x$

Left-hand side $= \cos(2x + x) =$

4. Given $\sin\alpha = \frac{12}{13}$, $\alpha$ is in Quadrant II, $\cos\beta = -\frac{4}{5}$, and $\beta$ is in Quadrant III, find $\sin(\alpha+\beta)$.

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$



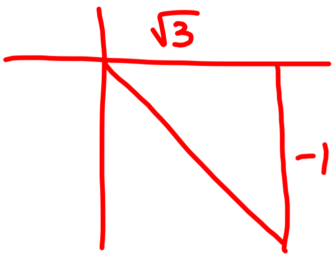

$$= \left(\frac{12}{13}\right)\left(\frac{-4}{5}\right) + \left(\frac{-5}{13}\right)\left(\frac{-3}{5}\right)$$

$$= \frac{-48}{65} + \frac{15}{65}$$

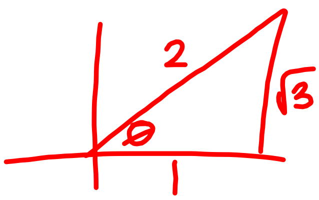
$$= \boxed{\frac{-33}{65}}$$

5. Find $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ exactly in radians.

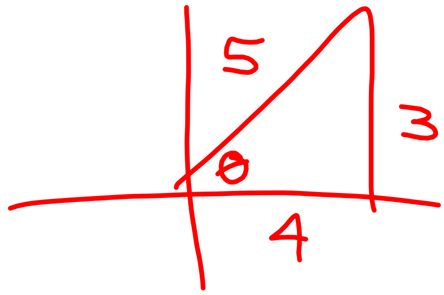
$$= \boxed{\frac{-\pi}{6}}$$



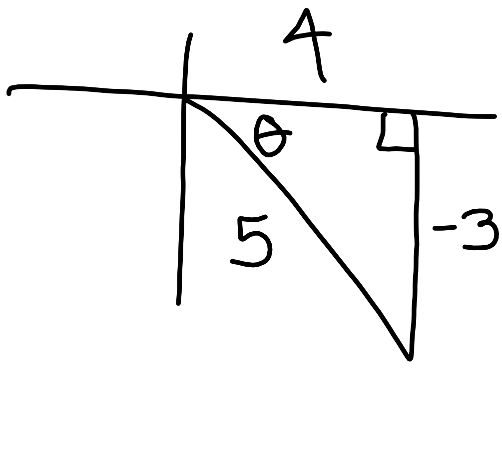

6. Evaluate $\cos\left(\csc^{-1}\frac{2}{\sqrt{3}}\right) = \cos\frac{\pi}{3} = \boxed{\frac{1}{2}}$



$$\cos\left[\csc^{-1}\left(\frac{5}{3}\right)\right] = \boxed{\frac{4}{5}}$$




$$\sec\left(\underbrace{\sin^{-1}\left(-\frac{3}{5}\right)}_{\theta}\right) = \boxed{\frac{5}{4}}$$



7. Find all solutions (in radians) in the interval $0 \le x < 2\pi$. $\sin^2 \overset{3x}{\cancel{x}} - \frac{1}{4} = 0$

$\sin^2(3x) - \frac{1}{4} = 0$

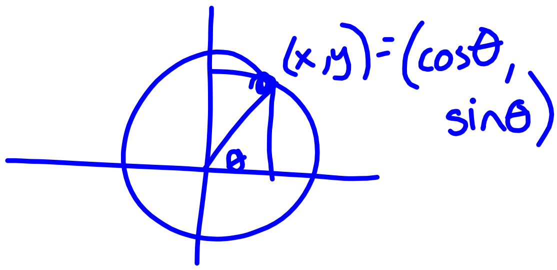
$\sin^2(3x) = \frac{1}{4}$

$\sin(3x) = \pm\frac{1}{2}$

$3x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}; \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}; \frac{25\pi}{6}, \frac{29\pi}{6}, \frac{31\pi}{6}, \frac{35\pi}{6}$

$\boxed{x = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{7\pi}{18}, \frac{11\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \frac{19\pi}{18}, \frac{23\pi}{18}, \frac{25\pi}{18}, \frac{29\pi}{18}, \frac{31\pi}{18}, \frac{35\pi}{18}}$

$\sin x + 1 = \cos x$




$(\sin x - \cos x)^2 = (-1)^2$

$\sin^2 x - 2\sin x\cos x + \cos^2 x = 1$

$(\sin^2 x + \cos^2 x) - 2\sin x\cos x = 1$

$1 - 2\sin x\cos x = 1$

$-2\sin x\cos x = 0$

$\sin x\cos x = 0$

$\sin x = 0, \cos x = 0$

$x = 0, \cancel{\frac{\pi}{2}}, \cancel{\pi}, \frac{3\pi}{2}$ ✓ ✓

$\boxed{x = 0, \frac{3\pi}{2}}$

8. Find all solutions (in radians) in the interval $0 \le x < 2\pi$. $2\sin^3 x = \sin x$

$2\sin^3 x - \sin x = 0$

$\sin x\,(\underbrace{2\sin^2 x - 1}_{-\cos 2x}) = 0$

$\sin x = 0$

$\boxed{x = 0, \pi}$

$2\sin^2 x - 1 = 0$

$\sin^2 x = \frac{1}{2}$

$\sin x = \pm\frac{1}{\sqrt{2}}$

$\boxed{x = \pi/4,\ 3\pi/4,\ 5\pi/4,\ 7\pi/4}$

9. Prove the identity.

$$\frac{1+\cos^2 x}{\sin^2 x} = 2\csc^2 x - 1$$

$LHS = \frac{1+\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} =$

$= \csc^2 x + \cot^2 x = \csc^2 x + (\csc^2 x - 1)$

$= 2\csc^2 x - 1$

$= RHS$ ✓

$\frac{\sin^2 x + \cos^2 x = 1}{\sin^2 x}$

$\cancel{1+}\cot^2 x = \csc^2 x - 1$

9. Prove the identity.

$$\frac{1+\cos^2 x}{\sin^2 x} = 2\csc^2 x - 1$$

$$RHS = \frac{2}{\sin^2 x} - 1 = \frac{2}{\sin^2 x} - \frac{\sin^2 x}{\sin^2 x}$$

$$= \frac{2 - \sin^2 x}{\sin^2 x} = \frac{2 - (1 - \cos^2 x)}{\sin^2 x}$$

$$= \frac{1 + \cos^2 x}{\sin^2 x} = LHS$$

10. Prove the identity. $\csc x - \cos x \cot x = \sin x$

$$LHS = \frac{1}{\sin x} - \frac{\cos x}{1} \cdot \frac{\cos x}{\sin x}$$

$$= \frac{1 - \cos^2 x}{\sin x} = \frac{\sin^2 x}{\sin x} = \sin x = RHS$$

Find all solutions (in radians) in the interval $0 \le x < 2\pi$.

$$\sin 3x + \sin x - \sin 2x = 0$$

$\sin(2x+x) + \sin x - \sin 2x = 0$

$\sin 2x \cos x + \cos 2x \sin x + \sin x - \sin 2x = 0$

$2\sin x\cos x\cos x + (1-2\sin^2 x)\sin x + \sin x - 2\sin x\cos x = 0$

$2\sin x\cos^2 x + \sin x - 2\sin^3 x + \sin x - 2\sin x\cos x = 0$

$2\sin x(1-\sin^2 x) + \sin x - 2\sin^3 x + \sin x - 2\sin x\cos x = 0$

$2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x + \sin x - 2\sin x\cos x = 0$

$-4\sin^3 x + 4\sin x - 2\sin x\cos x = 0$

$-2\sin x\left(2\sin^2 x - 2 + \cos x\right) = 0$

$-2\sin x\left(2(1-\cos^2 x) - 2 + \cos x\right) = 0$

$-2\sin x\left[\cancel{2} - 2\cos^2 x - \cancel{2} + \cos x\right] = 0$

$(-2\sin x)(\cos x)(-2\cos x + 1) = 0$

$-2\sin x = 0$ $\qquad -2\cos x + 1 = 0$

$\qquad\qquad\qquad -2\cos x = -1$

$\sin x = 0$, $\cos x = 0$, $\cos x = \frac{1}{2}$

$$\boxed{x = 0, \pi, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}, \frac{5\pi}{3}}$$

## **Homework for Test #3**:

Homework #6 (submitted Wed. 09/17)
- 6.2 #1-41 odd sum, difference, and cofunction identities

Homework #7 (submitted 09/26)
- 6.1 #1-69 odd proofs
- 6.3 #1-24 all double- and half-angle identities (application & proof)
  #30-36 all
  #49-93 odd
- 6.5 #1-24 all inverse functions
  #25-55 odd inverse functions

**Homework #8 (due Friday 10/03)**
- 6.6 #1-21 odd finding solutions between 0 and 2pi
- 6.6 #61-69 odd finding all possible solutions (+2pi*k)
- 6.6 #71-83 odd;
- Examples #3,4,7,8 from solving equations handout
- Test 3 Practice Problems handout

**Test #3 - Friday 10/3**