Review

$r = ?\ \text{in}\ ;\ s = 3\ \text{mi}\ ;\ \theta = 2640\ \text{rev}$

$s = r\theta$

$$r = \frac{s}{\theta} = \frac{3\ \cancel{\text{mi}}}{\cancel{2640}\ \cancel{\text{rev}}} \cdot \frac{1\ \cancel{\text{rev}}}{\cancel{2}\pi} \cdot \frac{\overset{2}{\cancel{5280}}\ \cancel{\text{ft}}}{1\ \cancel{\text{mi}}} \cdot \frac{12\ \text{in}}{1\ \cancel{\text{ft}}} = \boxed{\frac{36}{\pi}\ \text{in}}$$

$r = 3\ \text{ft}\ ;\ \theta = ?^\circ\ ;\ s = 4\ \text{in}$

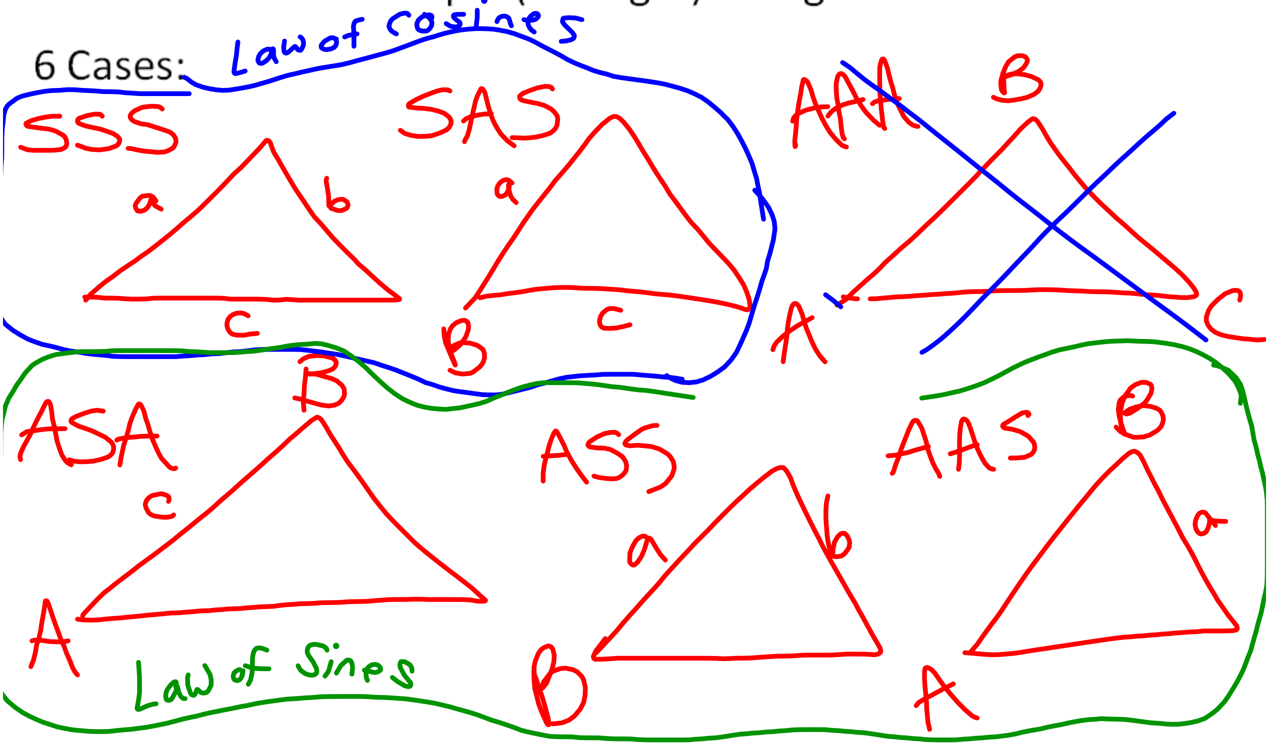
$s = r\theta$

$$\theta = \frac{s}{r} = \frac{\cancel{4}\ \cancel{\text{in}}}{\cancel{3}\ \cancel{\text{ft}}} \cdot \frac{1\ \cancel{\text{ft}}}{\underset{3}{\cancel{12}}\ \cancel{\text{in}}} \cdot \frac{\overset{20}{\cancel{180}}^\circ}{\pi} = \boxed{\frac{20}{\pi}^\circ}$$

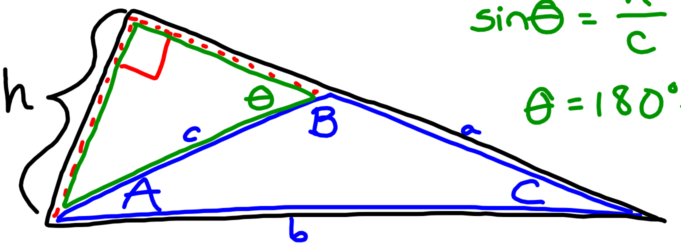
7.1 The Law of Sines

How do we solve oblique (not right) triangles?

6 Cases:



Derivation of the Law of Sines



$\sin\theta = \frac{h}{c}$ $\sin C = \frac{h}{b}$

$\theta = 180° - B$

$\sin B = \frac{h}{c}$ & $\sin C = \frac{h}{b}$

$c \cdot \sin B = h$ & $b\sin C = h$

$\sin(180° - B) = \sin 180° \cos B - \cos 180° \sin B$

$= 0 \cdot \cos B - (-1)\sin B$

$\sin(180° - B) = \sin B$

$$\frac{\cancel{c}\sin B}{b\cancel{c}} = \frac{\cancel{b}\sin C}{\cancel{b}c} \Rightarrow \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{c\cancel{\sin B}}{\cancel{\sin B}\sin C} = \frac{b\cancel{\sin C}}{\sin B\cancel{\sin C}} \Rightarrow \frac{c}{\sin C} = \frac{b}{\sin B}$$

The Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

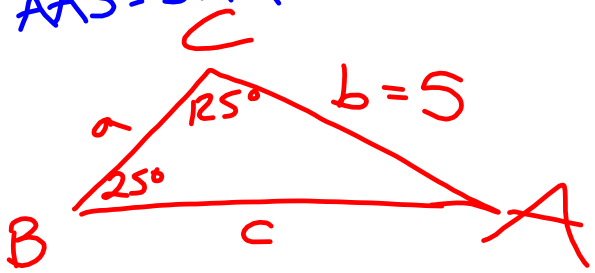
or

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

7.1

2. $B=25°, C=125°, b=5$

AAS = SAA



$A=180°-B-C$

$=180°-125°-25°$

$\boxed{A=30°}$

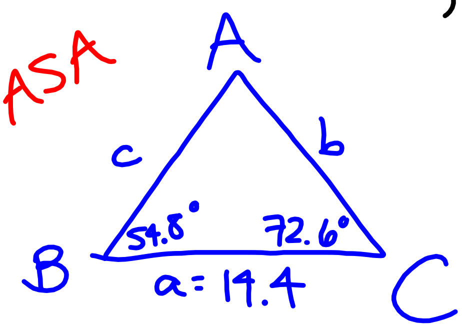
$\frac{a}{\sin A}=\frac{b}{\sin B}$

$a=\frac{b\sin A}{\sin B}=\frac{5\cdot\sin 30°}{\sin 25°}=\boxed{5.9=a}$

$\frac{c}{\sin C}=\frac{b}{\sin B}$ $c=\frac{b\cdot\sin C}{\sin B}=\frac{5\sin 125°}{\sin 25°}=\boxed{9.7=c}$

8. $B=54.8°, C=72.6°, a=14.4$

ASA



$A=180°-B-C$

$=180°-54.8°-72.6°=\boxed{52.6°=A}$

$\frac{b}{\sin B}=\frac{a}{\sin A}\Rightarrow b=\frac{a\sin B}{\sin A}$

$b=\frac{14.4\sin 54.8°}{\sin 52.6°}$

$\boxed{b=14.8}$

$\frac{c}{\sin C}=\frac{a}{\sin A}$

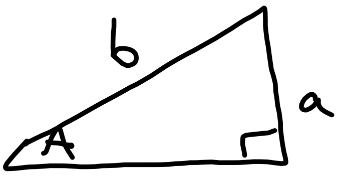
$c=\frac{a\sin C}{\sin A}=\frac{14.4\sin 72.6°}{\sin 52.6°}$

$\boxed{c=17.3}$

ASS, The Problematic Triangle

one solution:

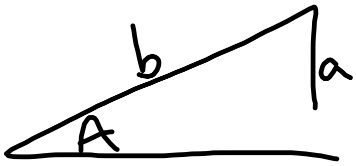
$a > b$



$a < b$

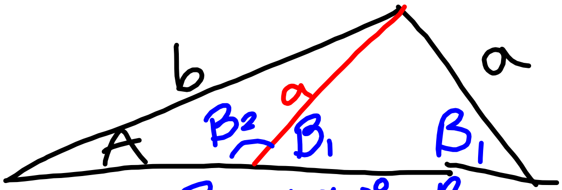
$B = 90^\circ$

no solutions:



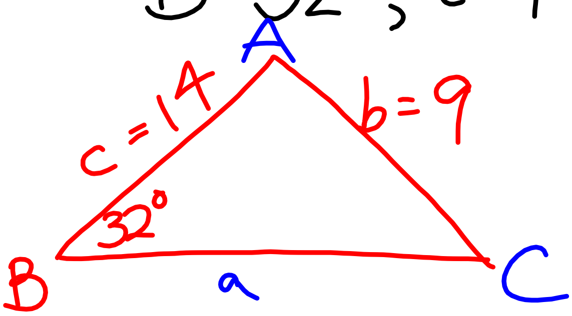
$a < b$

two solutions: $a < b$



$B_2 = 180^\circ - B_1$

14. $B = 32^\circ,\ c = 14,\ b = 9$



ASS, $b < c \Rightarrow$ ~~0~~, ~~1~~, or 2 solutions

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\sin C = \frac{c \cdot \sin B}{b}$$

$$\sin^{-1}(\sin C) = \sin^{-1}\left(\frac{c \cdot \sin B}{b}\right)$$

$$C = \sin^{-1}\left(\frac{14 \sin 32^\circ}{9}\right)$$

$$\boxed{C_1 = 55.5^\circ}$$

Case 1

$$\boxed{C_1 = 55.5^\circ}$$

$$A_1 = 180^\circ - 32^\circ - 55.5^\circ$$

$$\boxed{A_1 = 92.5^\circ}$$

$$\frac{a_1}{\sin 92.5^\circ} = \frac{9}{\sin 32^\circ}$$

$$a_1 = \frac{9 \sin 92.5^\circ}{\sin 32^\circ}$$

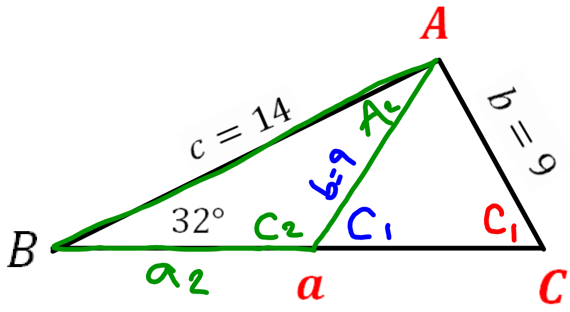
$$\boxed{a_1 = 17.0}$$

7.1 The Law of Sines, continued

ASS – Problematic Triangle

14. $B = 32°$, $c = 14$, $b = 9$

Case 1: $C \approx 55.5°$, $A \approx 92.5°$, $a \approx 17$



Case2

$C_2 = 180° - C_1 = 180° - 55.5° = \boxed{124.5° = C_2}$

$A_2 = 180° - B - C_2 = 180° - 32° - 124.5° = \boxed{23.5°}$

$\frac{a_2}{\sin A_2} = \frac{b}{\sin B}$

$a_2 = \frac{9 \cdot \sin 23.5°}{\sin 32°} = \boxed{6.8 = a_2}$

Homework:

- 7.1 #1-21 odd — solving triangles with Law of Sines
- 7.1 #29,30,33,34,35 — word problems with Law of Sines
- 7.2 #9-19 odd — solving triangles with Law of Cosines
- 7.2 #25-29odd; — area
- 7.2 #38,43,46,47,48 — word problems with Law of Cosines
- 7.3 #37,41,43 — word problems with Law of Sines/Cosines