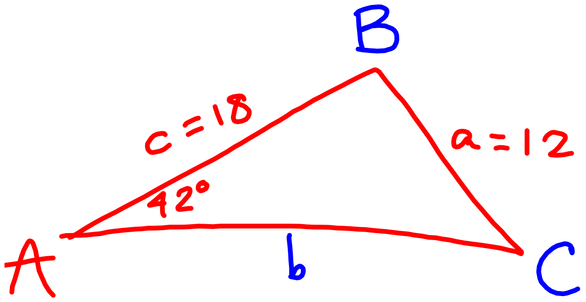
16. $A = 42°, a = 12, c = 18$



ASS triangle

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\sin C = \frac{c \cdot \sin A}{a}$$

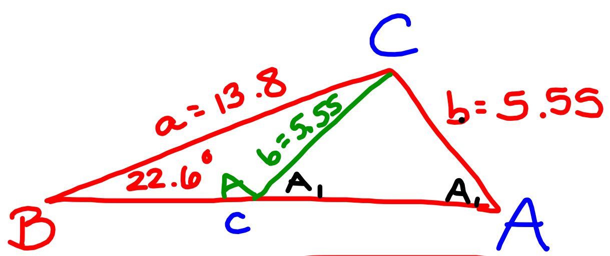
$$C = \sin^{-1}\left(\frac{c \cdot \sin A}{a}\right)$$

$$C = \sin^{-1}\left(\underbrace{\frac{18 \sin 42°}{12}}_{>1}\right)$$

undefined

$\boxed{\text{no such triangle exists}}$

18. $B = 22.6°, b = 5.55, a = 13.8$



Case 1

$$\frac{\sin A}{13.8} = \frac{\sin 22.6°}{5.55}$$

$$A = \sin^{-1}\left(\frac{13 \sin 22.6°}{5.55}\right)$$

$= \boxed{72.9°}$

$C = 180° - 22.6° - 72.9°$

$= \boxed{84.5°}$

$$\frac{c}{\sin 84.5°} = \frac{5.55}{\sin 22.6°}$$

$$c = \frac{5.55 \sin 84.5°}{\sin 22.6°}$$

$= \boxed{14.4}$

Case 2

$A = 180° - 72.9°$

$= \boxed{107.1°}$

$C = 180° - 107.1° - 22.6°$

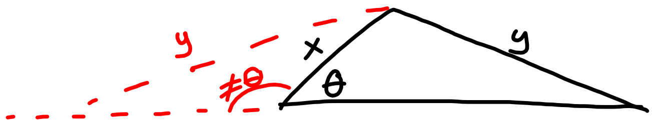
$= \boxed{50.3°}$

$$\frac{c}{\sin 50.3°} = \frac{5.55}{\sin 22.6°}$$

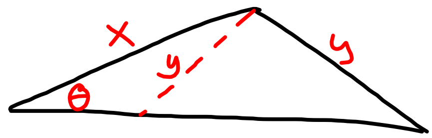
$$c = \frac{5.55 \sin 50.3°}{\sin 22.6°}$$

$= \boxed{11.1}$

Why does this ASS triangle have only one solution?



The measure of θ and the lengths of x & y are fixed. If we try to reposition y, the measure of θ changes, unlike in the 2-solution case:



7.2 - The Law of Cosines

Derivation:

x_1 y_1

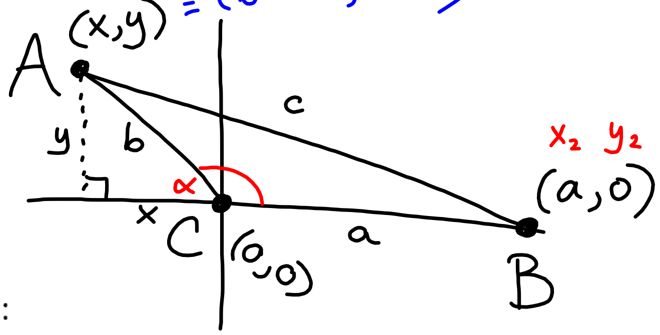
$(x,y) = (b\cos C, b\sin C)$

$\cos C = \frac{x}{b}$

$x = b\cos C$

$\sin C = \frac{y}{b}$

$y = b\sin C$



Distance Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$(x-y)^2 = x^2 - 2xy + y^2$

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$c^2 = (a - b\cos C)^2 + (0 - b\sin C)^2$$

$$c^2 = a^2 - 2ab\cos C + b^2\cos^2 C + b^2\sin^2 C$$

$$c^2 = a^2 + b^2(\underbrace{\sin^2 C + \cos^2 C}_{=1}) - 2ab\cos C$$

$$\boxed{c^2 = a^2 + b^2 - 2ab\cos C}$$

The Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\operatorname{Cos} A$$
$$b^2 = a^2 + c^2 - 2ac\operatorname{Cos} B$$
$$c^2 = a^2 + b^2 - 2ab\operatorname{Cos} C$$

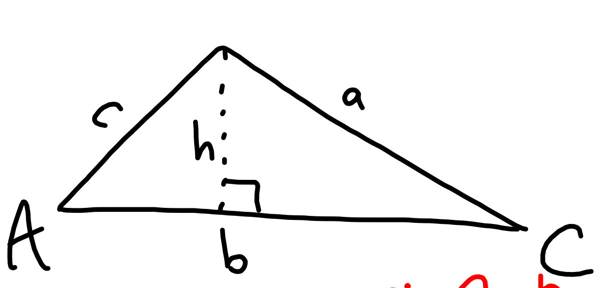
7.2

16. $a = 60, b = 88, c = 120.\ B = ?$

SSS ⇒ law of cosines

$$b^2 = a^2 + c^2 - 2ac\operatorname{Cos} B$$
$$2ac\operatorname{Cos} B = a^2 + c^2 - b^2$$
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$
$$B = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right) = \cos^{-1}\left(\frac{60^2 + 120^2 - 88^2}{2(60)(120)}\right) = \boxed{44.6^\circ}$$

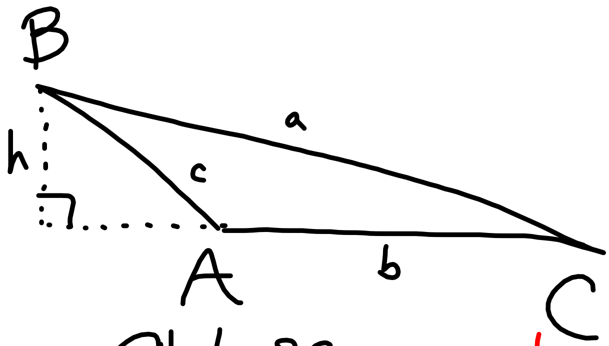
7.1/7.2 Area of a Triangle



Acute

$\sin C = \frac{h}{a}$

$h = a\sin C$



Obtuse

$\sin C = \frac{h}{a}$

$h = a\sin C$

$$\text{Area} = \frac{1}{2}(\text{base})(\text{height})$$
$$= \frac{1}{2}ab\sin C$$
$$= \frac{1}{2}ac\sin B$$
$$= \frac{1}{2}bc\sin A$$

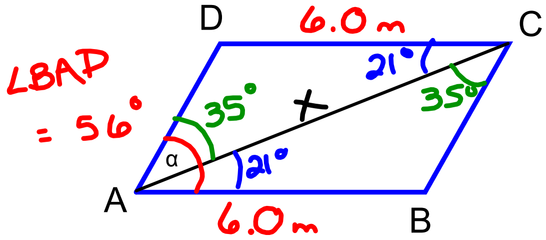
Find the area of the triangle.

$A = 50°, b = 13\ cm, c = 6\ cm$

$$\text{area} = \frac{1}{2}bc\sin A$$
$$= \frac{1}{2}(13)(6)\sin 50°$$
$$= 29.9\ cm^2$$

<u>7.1 #28</u>

The longer side of a parallelogram is 6.0 meters. The measure of angle BAD is 56° and α is 35°. Find the length of the longer diagonal.

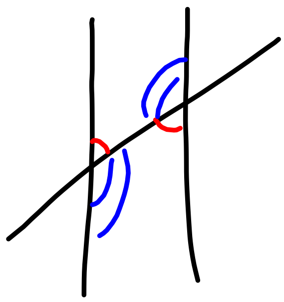


$$\angle D = 180° - 35° - 21°$$
$$= 124°$$

$$\frac{x}{\sin 124°} = \frac{6}{\sin 35°}$$

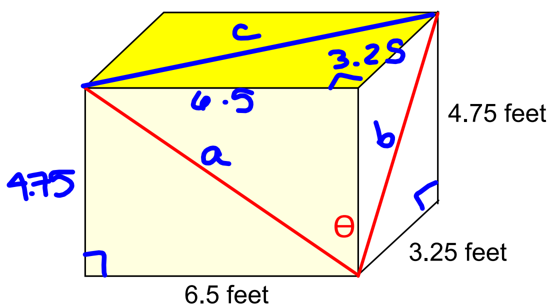
$$x = \frac{6 \sin 124°}{\sin 35°}$$

$$= \boxed{8.7\text{m}}$$



<u>7.2 #41</u>

The rectangular box in the figure measures 6.50 feet by 3.25 feet by 4.75 feet. Find the measure of the angle θ that is formed by the union of the diagonal shown on the front of the box and the diagonal shown on the right side of the box.



$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

$$a = \sqrt{6.5^2 + 4.75^2} =$$

$$b = \sqrt{3.25^2 + 4.75^2} =$$

$$c = \sqrt{6.5^2 + 3.25^2} =$$

Homework:

- 7.1 #1-21 odd — solving triangles with Law of Sines
- 7.1 #29,30,33,34,35 — word problems with Law of Sines
- 7.2 #9-19 odd — solving triangles with Law of Cosines
- 7.2 #25-29odd; — area
- 7.2 #38,43,46,47,48 — word problems with Law of Cosines
- 7.3 #37,41,43 — word problems with Law of Sines/Cosines