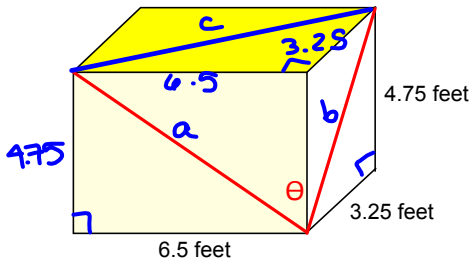
Word Problems with the Law of Sines and Cosines

7.2 #41

The rectangular box in the figure measures 6.50 feet by 3.25 feet by 4.75 feet. Find the measure of the angle Θ that is formed by the union of the diagonal shown on the front of the box and the diagonal shown on the right side of the box.



$c^2 = a^2 + b^2 - 2ab\cos\theta$

$2ab\cos\theta = a^2 + b^2 - c^2$

$\cos\theta = \frac{a^2 + b^2 - c^2}{2ab}$

$\theta = \cos^{-1}\left(\frac{a^2 + b^2 - c^2}{2ab}\right)$

$a = \sqrt{6.5^2 + 4.75^2} = 8.05$

$b = \sqrt{3.25^2 + 4.75^2} = 5.76$

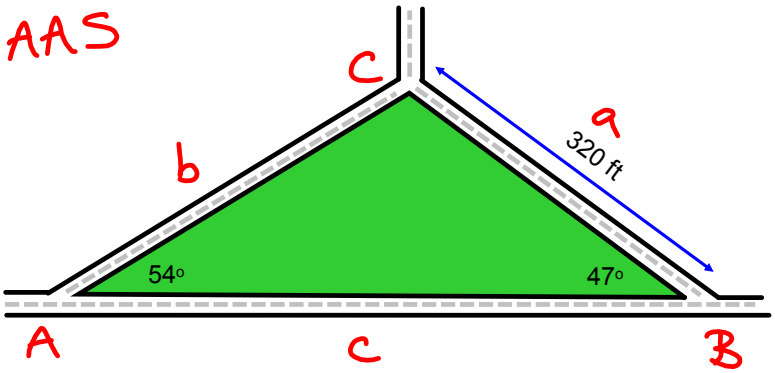
$c = \sqrt{6.5^2 + 3.25^2} = 7.27$

$= \cos^{-1}\left(\frac{8.05^2 + 5.76^2 - 7.27^2}{2(8.05)(5.76)}\right)$

$= 60.88°$

7.1 #32

Three roads intersect in such a way as to form a triangular piece of land. Find the lengths of the other two sides of the land.



$C = 180° - 54° - 47°$

$= 79°$

$\frac{b}{\sin 47°} = \frac{320}{\sin 54°}$

$b = \frac{320\sin 47°}{\sin 54°} = 289.3 \text{ ft}$

$\approx 290 \text{ ft}$

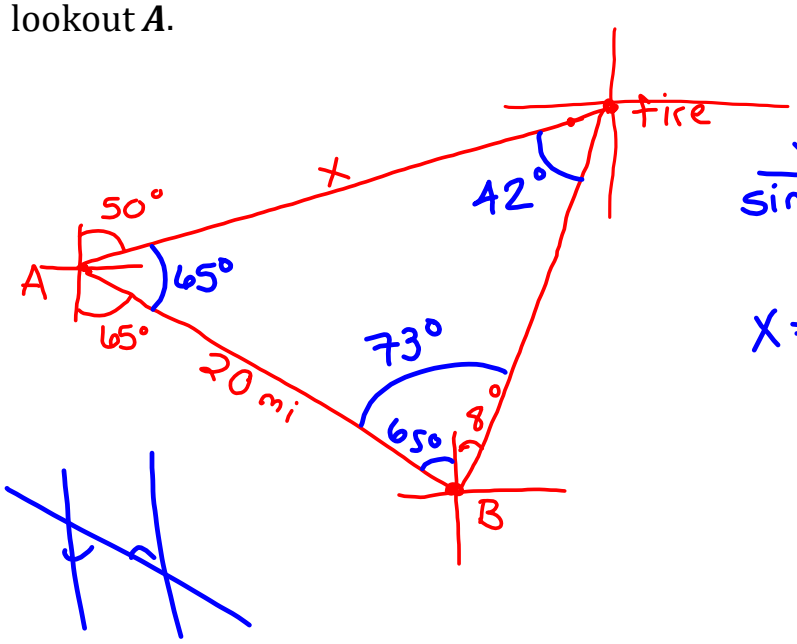
$\frac{c}{\sin 79°} = \frac{320}{\sin 54°}$

$c = \frac{320\sin 79°}{\sin 54°} =$

$= 388.3 \text{ ft} \approx 390 \text{ ft}$

7.1 #34

Two fire lookouts are located on mountains 20 miles apart. Lookout ***B*** is at a bearing of S65°E from ***A***. A fire was sighted at a bearing of N50°E from ***A*** and at a bearing of N8°E from ***B***. Find the distance of the fire from lookout ***A***.



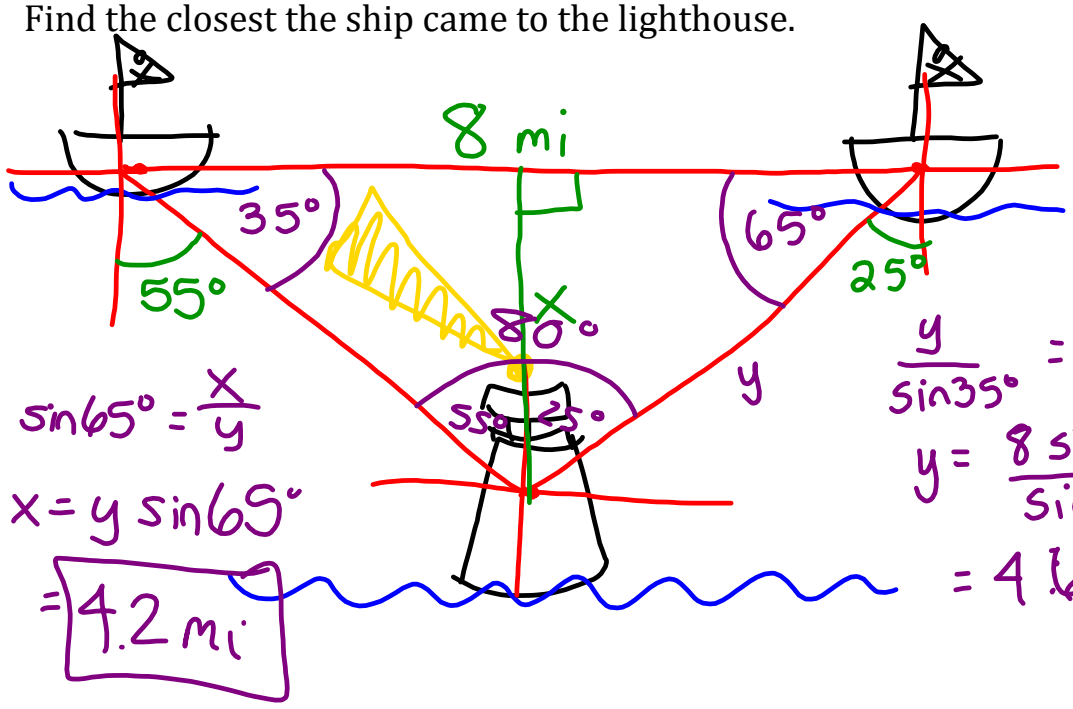
$$\frac{x}{\sin 73^\circ} = \frac{20}{\sin 42^\circ}$$

$$x = \frac{20 \sin 73^\circ}{\sin 42^\circ}$$

$$= 28.6 \text{ mi}$$

7.1 #36

The navigator on a ship traveling due east at 8 mph sights a lighthouse at a bearing of S55°E. One hour later it is sighted at a bearing of S25°W. Find the closest the ship came to the lighthouse.



$$\sin 65^\circ = \frac{x}{y}$$

$$x = y \sin 65^\circ$$

$$= 4.2 \text{ mi}$$

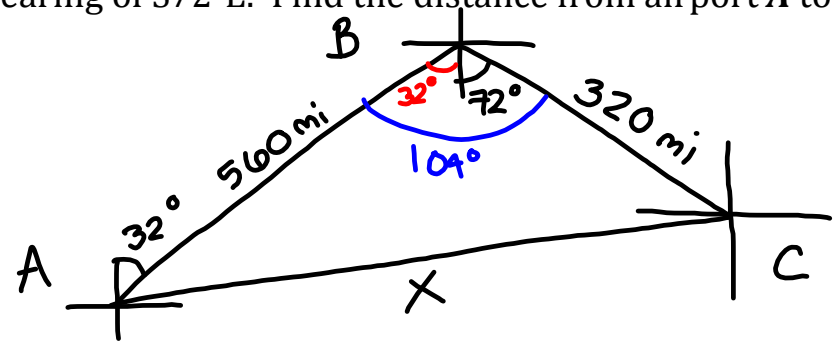
$$\frac{y}{\sin 35^\circ} = \frac{8}{\sin 80^\circ}$$

$$y = \frac{8 \sin 35^\circ}{\sin 80^\circ}$$

$$= 4.65 \text{ mi}$$

7.2 #37

A plane leaves airport ***A*** and travels 560 miles to airport ***B*** at a bearing of N32°E. The plane leaves airport ***B*** and travels to airport ***C*** 320 miles away at a bearing of S72°E. Find the distance from airport ***A*** to airport ***C***.



Law of Cosines:

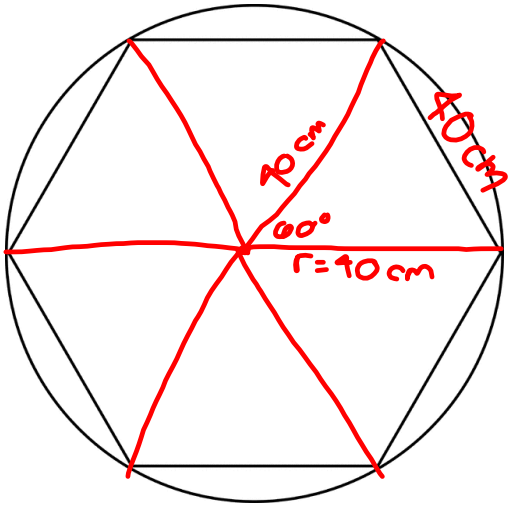
$$x^2 = 560^2 + 320^2 - 2(560)(320)\cos 104°$$

$$x = \sqrt{560^2 + 320^2 - 2(560)(320)\cos 104°}$$

$$\approx 709 \text{ mi}$$

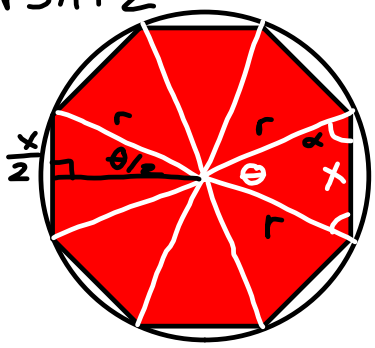
7.2 #45

A regular hexagon is inscribed in a circle with a radius of 40 centimeters. Find the length of one side of the hexagon.



$\sin\frac{\theta}{2} = \frac{x}{2}/r$

$x = 2r\sin\frac{\theta}{2}$



$\theta = \frac{360°}{\#\text{ of sides of polygon}}$

$= \frac{360°}{8} = 45°$

$180° - 45° = 135°$

$\alpha = \frac{135°}{2} = 67.5°$

Law of Sines:

$\frac{x}{\sin\theta} = \frac{r}{\sin\alpha}$

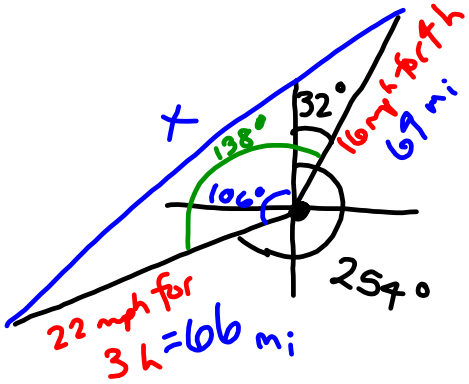
Law of Cosines:

$x = \sqrt{r^2 + r^2 - 2rr\cos\theta}$

7.2 #52

A ship leaves a port at a speed of 16 mph at a heading of 32°. One hour later another ship leaves the port at a speed of 22 mph at a heading of 254°. Find the distance between the ships 4 hours after the first ship leaves the port.

heading is always measured clockwise from North



$x = \sqrt{64^2 + 66^2 - 2(64)(66)\cos 138°}$

$= 121.4 \text{ mi}$

Homework:

- 7.1 #1-21 odd — solving triangles with Law of Sines
- 7.1 #29,30,33,34,35 — word problems with Law of Sines
- 7.2 #9-19 odd — solving triangles with Law of Cosines
- 7.2 #25-29odd; — area
- 7.2 #38,43,46,47,48 — word problems with Law of Cosines
- 7.3 #37,41,43 — word problems with Law of Sines/Cosines