stop @ 2:25 for quiz!

Vector multiplication

$\vec{v} \cdot \vec{w}$ vs. $\vec{v} \times \vec{w}$

dot product

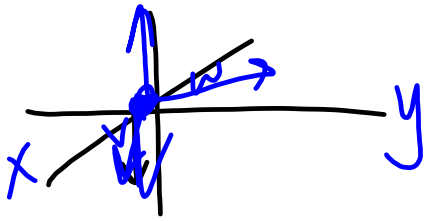
result is a scalar (#)

cross product

result is a vector

$\vec{v} = \langle a, b \rangle, \vec{w} = \langle c, d \rangle$

$\vec{v} \cdot \vec{w} = ac + bd$



stop @ 2:25 for quiz!

$\vec{v} = \langle 1, 2 \rangle$, $\vec{w} = \langle -3, 4 \rangle$

$\vec{v} \cdot \vec{w} = 1(-3) + 2(4) = -3 + 8 = \boxed{5}$

$\vec{v}_1 = \langle 1, 2 \rangle$, $\vec{v}_2 = \langle -3, 4 \rangle$, $\vec{v}_3 = \langle 5, -6 \rangle$

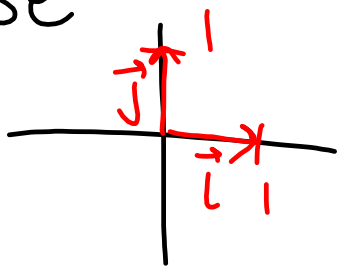
$\vec{v}_1 \cdot (\vec{v}_2 + \vec{v}_3) = \langle 1, 2 \rangle \cdot \langle 2, -2 \rangle$

$= 2 + (-4) = \boxed{-2}$

stop @ 2:25 for quiz!

Unit Vectors

A unit vector is a vector whose magnitude is 1.



special unit vectors:

$\vec{i} = \langle 1, 0 \rangle$ & $\vec{j} = \langle 0, 1 \rangle$

If 3 dimensions...

$\vec{i} = \langle 1, 0, 0 \rangle$; $\vec{j} = \langle 0, 1, 0 \rangle$; $\vec{k} = \langle 0, 0, 1 \rangle$

stop @ 2:25 for quiz!

$$\vec{v} = \langle a, b \rangle$$

$$= \langle a, 0 \rangle + \langle 0, b \rangle$$

$$= a\langle 1, 0 \rangle + b\langle 0, 1 \rangle$$

$$\vec{v} = a\vec{i} + b\vec{j}$$

$i = \sqrt{-1} \neq \vec{i}$

stop @ 2:25 for quiz!

$\vec{u} = 2\vec{i} + \vec{j}$; $\vec{v} = -3\vec{i} - 10\vec{j}$; $\vec{w} = \vec{i} - 5\vec{j}$

46. $\vec{v} + 3\vec{w} = -3\vec{i} - 10\vec{j} + 3(\vec{i} - 5\vec{j})$

$= -3\vec{i} - 10\vec{j} + 3\vec{i} - 15\vec{j}$

$= \boxed{-25\vec{j}} = \langle 0, -25 \rangle$

48. $(\vec{u} - \vec{v}) + \vec{w} = 2\vec{i} + \vec{j} - (-3\vec{i} - 10\vec{j}) + \vec{i} - 5\vec{j}$

$= \boxed{6\vec{i} + 6\vec{j}} = \langle 6, 6 \rangle$

stop @ 2:25 for quiz!

$\vec{v} = 3\vec{i} - 2\vec{j}$

$|\vec{v}| = \sqrt{3^2 + (-2)^2} = \sqrt{9 + 4} = \boxed{\sqrt{13}}$

common mistake:

$\sqrt{(3i)^2 + (-2j)^2}$

$= \sqrt{3i^2 + 4j^2}$

$= \sqrt{-3 \cdot 4} = \sqrt{-7}$

stop @ 2:25 for quiz!

Given a vector $\vec{v} = \langle a, b \rangle$ we can find a unit vector $\vec{u}$ in the direction of $\vec{v}$ by dividing each component by $|\vec{v}|$.

$$\vec{u} = \left\langle \frac{a}{\sqrt{a^2+b^2}}, \frac{b}{\sqrt{a^2+b^2}} \right\rangle$$

stop @ 2:25 for quiz!

$$\vec{v} = \langle -3, 4 \rangle$$

Find a unit vector $\vec{u}$ in the direction of $\vec{v}$.

$$|\vec{v}| = \sqrt{(-3)^2 + 4^2} = 5$$

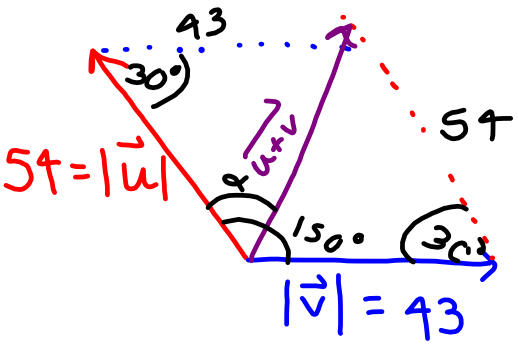
$$\vec{u} = \left\langle \frac{-3}{5}, \frac{4}{5} \right\rangle$$

$$|\vec{u}| = \sqrt{\left(\frac{-3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} = \sqrt{\frac{9}{25} + \frac{16}{25}} = \sqrt{\frac{25}{25}} = \sqrt{1} = 1 \checkmark$$

stop @ 2:25 for quiz!

~~Applications from section 7.5~~

18. Find $|\vec{u}+\vec{v}|$ and the angle that $\vec{u}+\vec{v}$ makes with $\vec{u}$, given $|\vec{u}|=54$, $|\vec{v}|=43$, & the angle θ between $\vec{u}$ & $\vec{v}$ is $150°$.



SAS:

$|\vec{u}+\vec{v}| = \sqrt{54^2+43^2-2\cdot54\cdot43\cdot\cos30°}$

$= 27.26$

$\frac{\sin\alpha}{43} = \frac{\sin 30°}{27.26}$

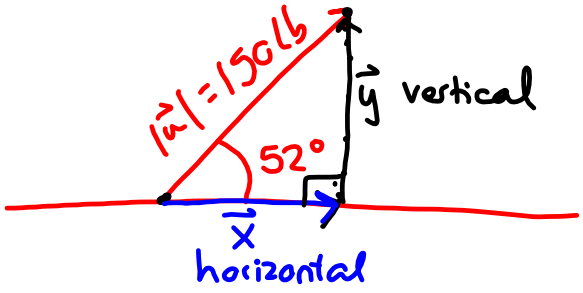
$\alpha = \sin^{-1}\left(\frac{43\sin30°}{27.26}\right)$

$= 52°$

stop @ 2:25 for quiz!

Resolving a vector into horizontal and vertical components

32. $|\vec{u}| = 150$ lb, inclined upward to the right at $52°$ from the horizontal. Resolve $\vec{u}$ into horizontal and vertical components.



$\cos 52° = \frac{x}{150}$

$\vec{x} = 150\cos52°$

$\sin 52° = \frac{y}{150}$

$\vec{y} = 150\sin52°$

$\vec{v} = \langle|\vec{v}|\cos\theta, |\vec{v}|\sin\theta\rangle$

$\vec{u} = \langle 150\cos52°, 150\sin52°\rangle$

Homework #9 (due Fri. 10/17):

- 7.1 #1-21 odd — solving triangles with Law of Sines
- 7.1 #29,30,33,34,35 — word problems with Law of Sines
- 7.2 #9-19 odd — solving triangles with Law of Cosines
- 7.2 #25-29odd; — area
- 7.2 #38,43,46,47,48 — word problems with Law of Cosines
- 7.3 #37,41,43 — word problems with Law of Sines/Cosines

Homework #10 (due Fri. 10/24)

- 7.3 #1-35 odd — vector operations
- 7.3 #45-59 odd — dot product and angle between vectors
- 7.4 #1-65 odd — trigonometric form of complex numbers

Test #4 - Wed. 10/22