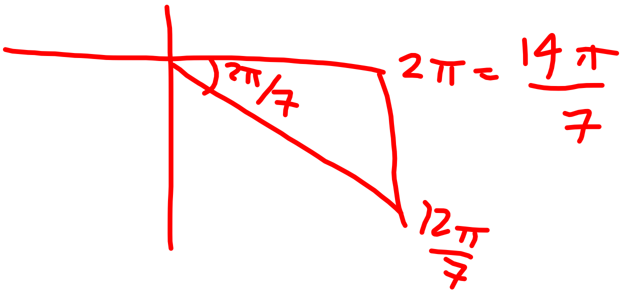
Review: Find the area of the given triangle. Your answer must include appropriate units.

7. $B = 42°$, $a = 7.2\ ft$, $c = 3.4\ ft$

$$\frac{1}{2}ac\cdot\sin B = \frac{1}{2}(7.2)(3.4)\sin 42° = \boxed{8.2\ ft^2}$$

Evaluate:

$$\cos^{-1}\left(\cos\frac{12\pi}{7}\right) = \boxed{\frac{2\pi}{7}}$$



Prove.

$$\underbrace{\cos^2x - 2\sin^2x\cos^2x} - \underbrace{\sin^2x + 2\sin^4x} = \cos^2 2x$$

$$LHS = \cos^2x\left(1-2\sin^2x\right) - \sin^2x\left(1-2\sin^2x\right) =$$

$$= \left(1-2\sin^2x\right)\left(\cos^2x - \sin^2x\right) =$$

$$= \cos 2x \cdot \cos 2x =$$

$$= \cos^2 2x =$$

$$= RHS$$

Solve for $x \in [0, 2\pi)$

$$2\cos^3 x = \cos x$$

$$2\cos^3 x - \cos x = 0$$

$$\cos x\,(2\cos^2 x - 1) = 0$$

$\cos x = 0$

$$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}}$$

$2\cos^2 x - 1 = 0$

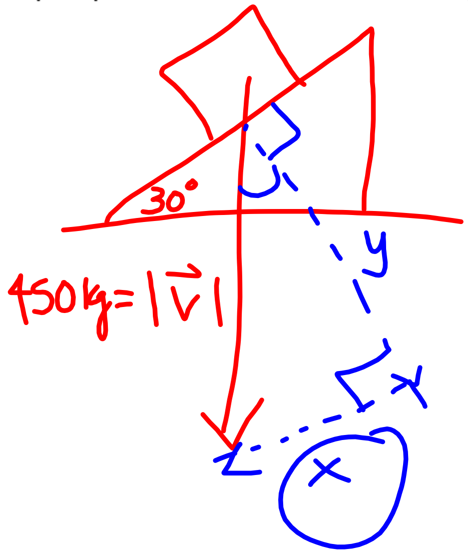
$$\cos^2 x = \frac{1}{2}$$

$$\cos x = \pm \frac{1}{\sqrt{2}}$$

$$\boxed{x = \pi/4,\ 3\pi/4,\ 5\pi/4,\ 7\pi/4}$$

The object on a ramp problem

40. If a 450kg object is at rest on a ramp with a 30° incline, find the components of the force of the object's weight parallel and perpendicular to the ramp. (bad physics)



parallel:

$$\sin 30^\circ = \frac{x}{450}$$

$$x = 450 \sin 30^\circ = \boxed{225\text{kg}}$$

perpendicular/normal

$$\cos 30^\circ = \frac{y}{450}$$

$$y = 450 \cos 30^\circ = 225\sqrt{3}\text{ kg} \approx 389.7\text{ kg}$$

Recall

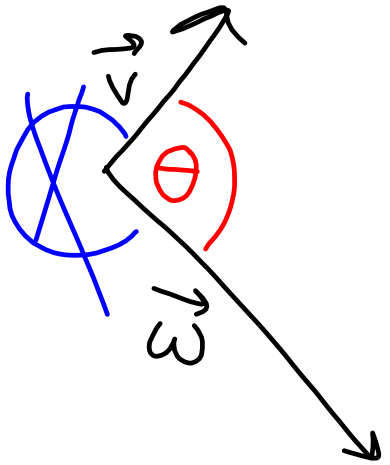
Given vectors $\vec{v} = \langle a, b \rangle$ & $\vec{w} = \langle c, d \rangle$,

the dot product of $\vec{v}$ & $\vec{w}$ is

$$\vec{v} \cdot \vec{w} = ac + bd$$

The smallest nonnegative angle between two vectors is given by

$\cos\theta = \frac{\vec{v}\cdot\vec{w}}{|\vec{v}||\vec{w}|}$, or equivalently, $\boxed{\theta = \cos^{-1}\frac{\vec{v}\cdot\vec{w}}{|\vec{v}||\vec{w}|}}$.



7.6

$$\boxed{\cos\theta = \frac{\vec{v}\cdot\vec{w}}{|\vec{v}||\vec{w}|}}$$

64. $\vec{a} = \langle -3, -3 \rangle$; $\vec{b} = \langle -5, 2 \rangle$

$\vec{a}\cdot\vec{b} = (-3)(-5) + (-3)(2) = 15 - 6 = 9$

$|\vec{a}| = \sqrt{(-3)^2 + (-3)^2} = \sqrt{18} = 3\sqrt{2}$

$|\vec{b}| = \sqrt{(-5)^2 + 2^2} = \sqrt{29}$

$$\theta = \cos^{-1}\left(\frac{9}{3\sqrt{2}\sqrt{29}}\right) = \boxed{66.8^\circ}$$

68. $\vec{u} = 3\vec{i} + 2\vec{j}$; $\vec{v} = -\vec{i} + 4\vec{j}$

$\vec{u}\cdot\vec{v} = 3(-1) + 2(4) = 5$

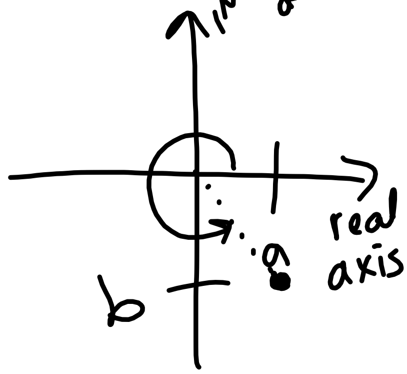
$|\vec{u}| = \sqrt{3^2 + 2^2} = \sqrt{13}$

$|\vec{v}| = \sqrt{(-1)^2 + 4^2} = \sqrt{17}$

$$\theta = \cos^{-1}\left(\frac{5}{\sqrt{13}\sqrt{17}}\right) = \boxed{70.3^\circ}$$

Trigonometric Form of Complex Numbers

z = a + bi , where i = $\sqrt{-1}$; $a, b \in \mathbb{R}$



a is the "real component"
b is the "imaginary component

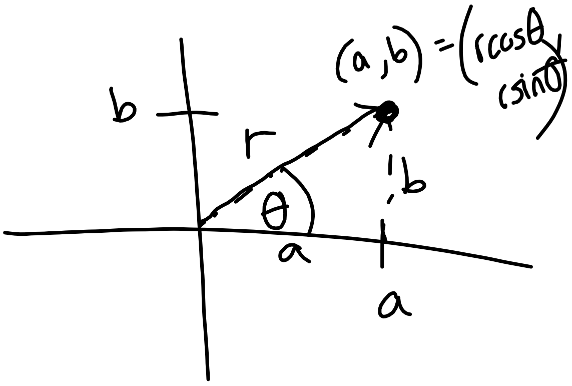
modulus $|z| = \sqrt{a^2+b^2}$

argument θ (counter-clockwise from 0)

r cis

mult/div

$$z = a + bi$$



$$z = r\cos\theta + r\sin\theta \cdot i$$

$$= r(\cos\theta + i\sin\theta)$$

$$z = r \operatorname{cis}\theta$$

$$\cos\theta = \frac{a}{r} \quad \sin\theta = \frac{b}{r}$$

$$a = r\cos\theta \quad b = r\sin\theta$$

$$z_1 = r_1 \operatorname{cis}\theta_1 \; ; \; z_2 = r_2 \operatorname{cis}\theta_2$$

$$z_1 z_2 = (r_1 \operatorname{cis}\theta_1)(r_2 \operatorname{cis}\theta_2) =$$

$$= (r_1\cos\theta_1 + i \cdot r_1 \sin\theta_1)(r_2\cos\theta_2 + i \cdot r_2 \sin\theta_2)$$

$$= r_1\cos\theta_1 r_2\cos\theta_2 + r_1\cos\theta_1 i\, r_2\sin\theta_2 + i\, r_1\sin\theta_1 r_2\cos\theta_2 + i\, r_1\sin\theta_1\, i\, r_2\sin\theta_2$$

$$= r_1 r_2 \cos\theta_1\cos\theta_2 + i^2 r_1 r_2 \sin\theta_1 \sin\theta_2 + i r_1 r_2 \cos\theta_1 \sin\theta_2 + i r_1 r_2 \sin\theta_1 \cos\theta_2$$

$$= r_1 r_2 (\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2) + i \cdot r_1 r_2 (\cos\theta_1 \sin\theta_2 + \sin\theta_1\cos\theta_2)$$

$$= r_1 r_2 \cos(\theta_1 + \theta_2) + i r_1 r_2 \sin(\theta_1 + \theta_2)$$

$$\therefore z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$$

$$z_1 = -2 \operatorname{cis} 20^\circ \; ; \; z_2 = 5 \operatorname{cis} 40^\circ$$

$$z_1 z_2 = (-2)(5)\operatorname{cis}(20^\circ + 40^\circ)$$

$$= \boxed{-10 \operatorname{cis} 60^\circ}$$

$$z_1 = r_1 \operatorname{cis}\theta_1 \; ; \; z_2 = r_2 \operatorname{cis}\theta_2$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

$$z_1 = 12 \operatorname{cis} 120^\circ \; ; \; z_2 = -4 \operatorname{cis} 10^\circ$$

$$\frac{z_1}{z_2} = \frac{12}{-4} \operatorname{cis}(120^\circ - 10^\circ) = \boxed{-3 \operatorname{cis} 110^\circ}$$

Extra Credit Opportunity TODAY:

5:00-5:30pm Mon 10/20 in S205 - Math League Competition

Test #4 - Wed. 10/22 (review, law of sines/cosines, area, basic vector operations and word problems; no dot product or complex numbers!)

Homework #10 (due Fri. 10/24)

- 7.3 #1-35 odd — vector operations
- 7.3 #45-59 odd — dot product and angle between vectors
- 7.4 #1-65 odd — trigonometric form of complex numbers

Quiz #8 - Fri. 10/24 (review, vector dot product, trig form of complex numbers)

Final Exam - 1:00pm, Tues. 10/28