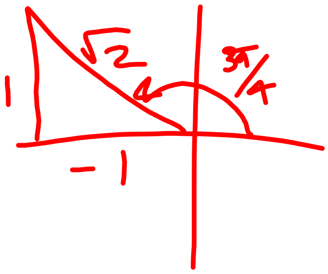
$$\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$$
$$\tan\frac{x}{2} = \frac{\sin x}{1+\cos x}$$
$$\tan\frac{x}{2} = \frac{1-\cos x}{\sin x}$$

Use the half-angle identity to evaluate $\tan\frac{3\pi}{8}$ exactly.

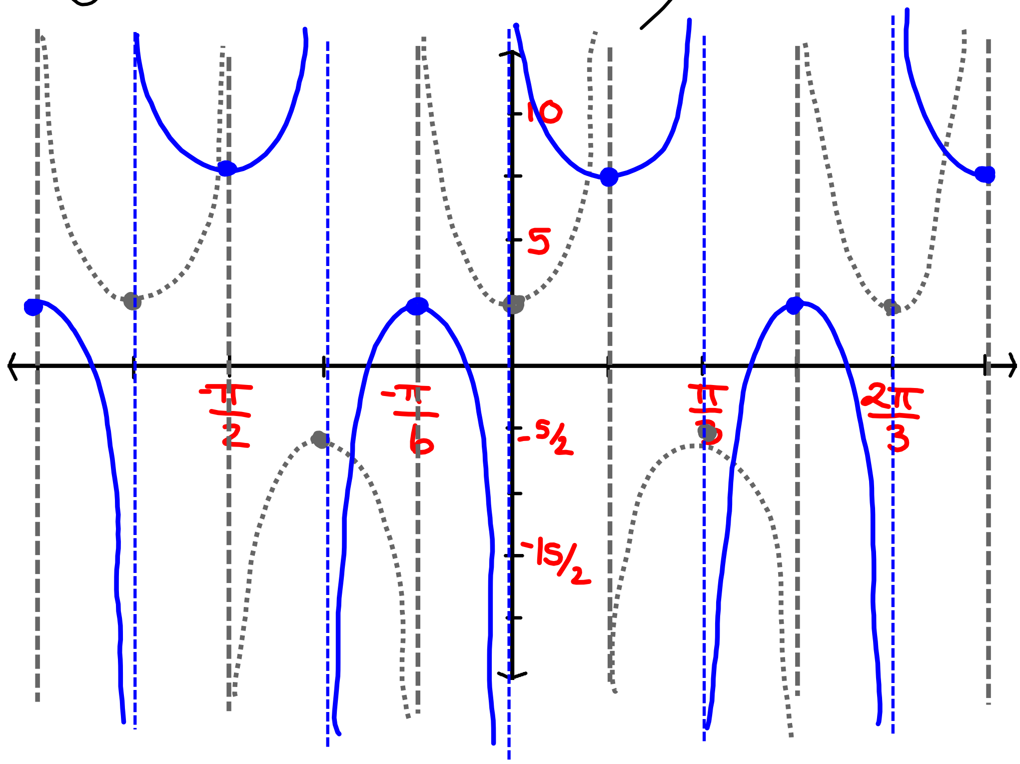
$$\tan\frac{3\pi}{8} = \tan\frac{3\pi/4}{2} = \frac{1-\cos\frac{3\pi}{4}}{\sin\frac{3\pi}{4}}$$



$$= \frac{1-\left(\frac{-\sqrt{2}}{2}\right)}{1/\sqrt{2}}$$

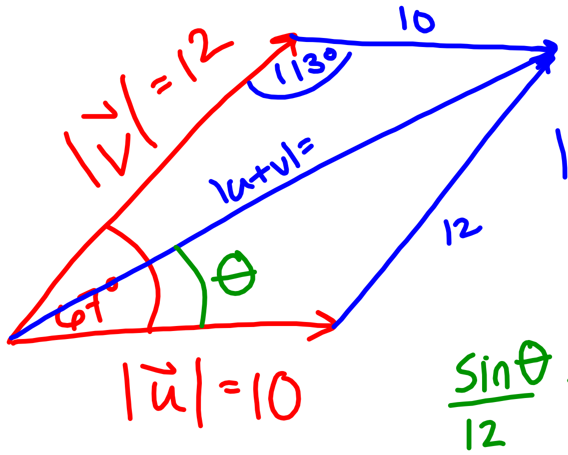
$$= \left(1+\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{1}\right) = \boxed{\sqrt{2}+1}$$

Graph: $y = \frac{5}{2}\sec\left(3x - \frac{\pi}{2}\right) + 5$



$|\vec{u}|=10,\ |\vec{v}|=12,\ \theta=67^\circ$

$|\overrightarrow{u+v}|=?$ & $\angle$ betw. $\overrightarrow{u+v}$ & $\vec{u}$

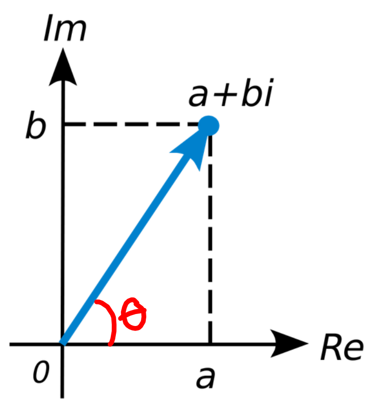


$$|\overrightarrow{u+v}| = \sqrt{10^2+12^2-2(10)(12)\cos 113^\circ}$$
$$=\boxed{18.4}$$

$$\frac{\sin\theta}{12} = \frac{\sin 113^\circ}{18.4}$$

$$\theta = \sin^{-1}\left(\frac{12\sin 113^\circ}{18.4}\right) = \boxed{36.9^\circ}$$

7.3 Trigonometric Form of Complex Numbers



Complex Number review:

$z = a + bi$, where $i = \sqrt{-1}$; $a, b \in \mathbb{R}$

a is the "real component"

b is the "imaginary component"

The modulus of a complex number is its distance from the origin.

$|z| = \sqrt{a^2 + b^2}$

The argument θ of a complex number is the direction angle, measured counter-clockwise from the positive x-axis.

Multiplying complex #'s in trigonometric form

$z_1 = r_1 cis\theta_1$; $z_2 = r_2 cis\theta_2$ $= r_2\cos\theta_2 + r_2 i\sin\theta_2$

$z_1 z_2 = r_1 r_2 \, cis(\theta_1 + \theta_2)$

To multiply two complex numbers in trigonometric form, multiply the moduli and add the arguments.

$z_1 z_2 \cdots z_n = r_1 r_2 \cdots r_n \, cis(\theta_1 + \cdots + \theta_n)$

$z^n = r^n cis(n\theta)$

Dividing complex #'s in trigonometric form

$z_1 = r_1 cis\theta_1$; $z_2 = r_2 cis\theta_2$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} cis(\theta_1 - \theta_2)$$

To divide two complex numbers in trigonometric form, divide the moduli and subtract the arguments.

Converting between

Standard Form & Trigonometric Form

$a + bi$ $\qquad\qquad$ $r\text{cis}\theta$

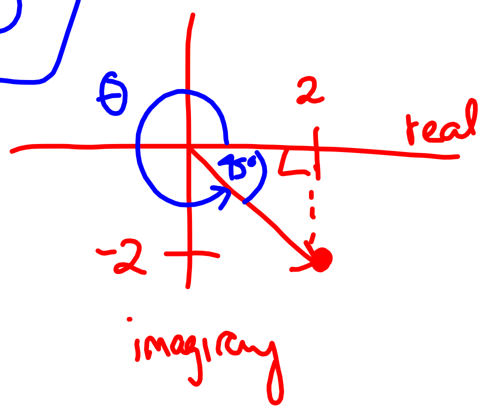
$$z = 5cis30° = 5\cos 30° + 5i\sin 30°$$

$$= \boxed{\frac{5\sqrt{3}}{2} + \frac{5}{2}i}$$

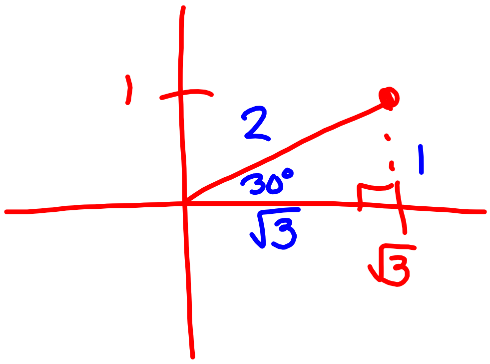
$$z = 2 - 2i = \boxed{2\sqrt{2}\,\text{cis}\,315°}$$

$$|z| = \sqrt{2^2 + (-2)^2}$$

$$= 2\sqrt{2}$$

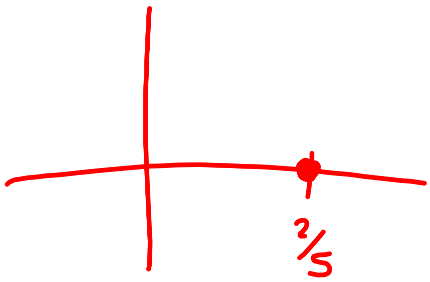


$\sqrt{3} + i = \boxed{2 \text{ cis } 30°}$



$\frac{2}{5} + 0i$

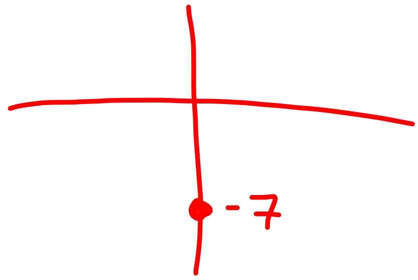
$= \boxed{\frac{2}{5} \text{ cis } 0°}$



$= \frac{2}{5}\left(\cos 0° + i \sin 0°\right)$

$0 - 7i$

$= \boxed{7\text{cis}270^\circ}$



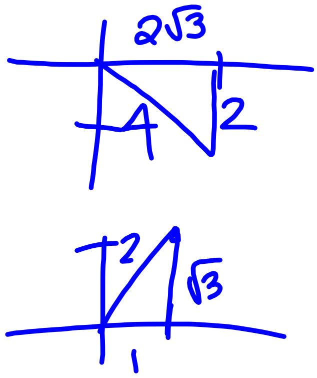
$$\frac{2\sqrt{3}-2i}{1+\sqrt{3}i} \cdot \frac{1-\sqrt{3}i}{1-\sqrt{3}i} = \frac{2\sqrt{3}-6i-2i+2\sqrt{3}i^2}{1-3i^2}$$

$$= \frac{2\sqrt{3}-8i-2\sqrt{3}}{1+3} = \frac{-8i}{4} = \boxed{-2i}$$

OR

$$\frac{2\sqrt{3}-2i}{1+\sqrt{3}i} = \frac{4\text{cis}330^\circ}{2\text{cis}60^\circ} = 2\text{cis}(270^\circ)$$

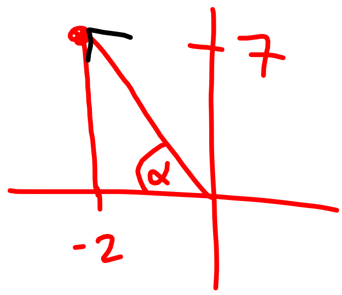
$$= 2\left(\cancel{\cos 270^\circ} + i\,\overset{-1}{\cancel{\sin 270^\circ}}\right) = \boxed{-2i}$$



Given a vector, determine the magnitude of the vector, the direction angle of the vector, and a unit vector in the same direction as that vector.

$\vec{v} = < -2, 7 >$

$|\vec{v}| = \sqrt{(-2)^2 + 7^2} = \boxed{\sqrt{53}}$



$\theta = 180° - \alpha = \boxed{105.9°}$

$\tan\alpha = \left|\frac{7}{2}\right|$

$\alpha = \tan^{-1}\left(\frac{7}{2}\right)$

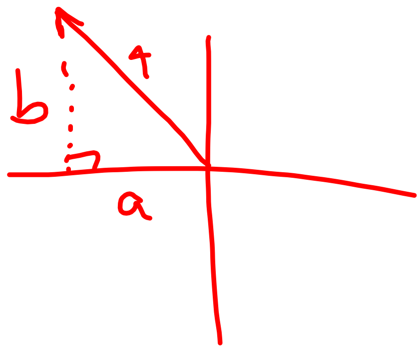
$= 74.1°$

$\vec{u} = \left\langle \frac{-2}{\sqrt{53}}, \frac{7}{\sqrt{53}} \right\rangle$

$= \boxed{\left\langle \frac{-2\sqrt{53}}{53}, \frac{7\sqrt{53}}{53} \right\rangle}$

$|\vec{v}| = 4$; $\theta = 127°$ $\langle a, b \rangle = ?$

$\langle 4\cos 127°, 4\sin 127° \rangle$

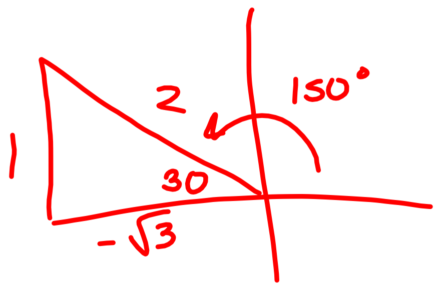


$= \boxed{\langle -2.4, 3.2 \rangle}$

$|\vec{v}| = 5 \quad ; \quad \theta = 150^\circ$

$\vec{v} = \langle 5\cos 150^\circ, 5\sin 150^\circ \rangle$

$= \boxed{\left\langle -\frac{5\sqrt{3}}{2}, \frac{5}{2} \right\rangle}$



[Homework #10](#) (due Fri. 10/24)

- 7.3 #1-35 odd vector operations
- 7.3 #45-59 odd dot product and angle between vectors
- 7.4 #1-65 odd trigonometric form of complex numbers

Quiz #8 - Fri. 10/24 (review, vector dot product, trig form of complex numbers)

[Homework #11](#) (due Monday 10/27)

- Final Exam Practice Problems

Review Session - 3:45pm, Monday, 10/27

Final Exam - 1:00pm, Tues. 10/28