Homework #10 (due Fri. 10/24)

- 7.3 #1-35 odd vector operations
- 7.3 #45-59 odd dot product and angle between vectors
- 7.4 #1-65 odd trigonometric form of complex numbers

Quiz #8 - Fri. 10/24 (review, vector dot product, trig form of complex numbers)

Homework #11 (due Monday 10/27)

- Final Exam Practice Problems

Review Session - 3:45pm, Monday, 10/27

Final Exam - 1:00pm, Tues. 10/28

Find the radius (in cm) of a circle that contains an arc of length 8m, subtending a 100° angle. $r = ?\,cm$; $s = 8m$; $\theta = 100°$

$s = r\theta$

$$r = \frac{s}{\theta} = \frac{8\cancel{m}}{\cancel{100°}} \cdot \frac{180\cancel{°}}{\pi} \cdot \frac{\cancel{100}\,cm}{1\cancel{m}} = \boxed{\frac{1440}{\pi}\ cm}$$

Solve for x. (all solutions, no restrictions)

$\sin 2x - \sin x = 0$

$2\sin x\cos x - \sin x = 0$

$\sin x(2\cos x - 1) = 0$

$\sin x = 0 \qquad \cos x = \frac{1}{2}$

$x = \pi k$

$x = \frac{\pi}{3} + 2\pi k$

$x = \frac{5\pi}{3} + 2\pi k$

Given $\vec{v} = \langle -1,6 \rangle$, $\vec{w} = \langle 6, -1 \rangle$

1. Find $2\vec{v} - 3\vec{w}$.

$\langle -2, 12 \rangle - \langle 18, -3 \rangle = \langle -20, 15 \rangle$

2. Find $|\vec{v}|$. $\sqrt{(-1)^2 + 6^2} = \sqrt{37}$

3. Find $|\vec{w}|$. $\sqrt{6^2 + (-1)^2} = \sqrt{37}$

4. Find $\vec{v} \cdot \vec{w}$.

$(-1)(6) + 6(-1) = -12$

5. Find the angle θ between $\vec{v}$ and $\vec{w}$.

$\cos^{-1}\left(\frac{\vec{v} \cdot \vec{w}}{|\vec{v}||\vec{w}|}\right) = \cos^{-1}\left(\frac{-12}{37}\right) = 108.9°$

6. Find a unit vector $\vec{u}$ in the same direction as $\vec{v}$.

$\left\langle \frac{-1}{\sqrt{37}}, \frac{6}{\sqrt{37}} \right\rangle$

Find a unit vector in the same direction as the given angle.

$\vec{r} = \langle -2, -8 \rangle$

$|\vec{r}| = \sqrt{(-2)^2 + (-8)^2} = \sqrt{68} = 2\sqrt{17}$

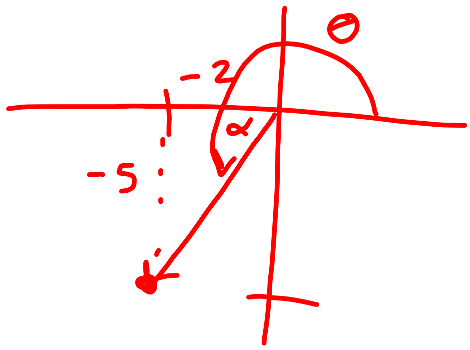
$\vec{u} = \left\langle \frac{-2}{2\sqrt{17}}, \frac{-8}{2\sqrt{17}} \right\rangle$

$= \left\langle \frac{-1}{\sqrt{17}}, \frac{-4}{\sqrt{17}} \right\rangle$

$= \left\langle \frac{-\sqrt{17}}{17}, \frac{-4\sqrt{17}}{17} \right\rangle$

Determine the direction angle of the given vector.

$\vec{u} = \langle -2, -5 \rangle$



$\alpha = \tan^{-1}\left(\frac{5}{2}\right)$

$= 68.2°$

$\theta = 180 + 68.2°$

$= \boxed{248.2°}$

Evaluate the inverse functions. Give your answers in radians.

1. $\csc^{-1}\left(-\frac{2}{\sqrt{3}}\right) = \boxed{-\frac{\pi}{3}}$

2. $\cot^{-1}(\sqrt{3}) = \boxed{\frac{\pi}{6}}$

$-\frac{\pi}{2}, \frac{\pi}{2}$: $\sin x, \csc x, \tan x$

$0, \pi$: $\cos x, \sec x, \cot x$

Evaluate.

1. $\sin^{-1}\left(\sin\frac{\pi}{5}\right) = \boxed{\frac{\pi}{5}}$

2. $\tan^{-1}\left(\tan\frac{2\pi}{3}\right) = \boxed{-\frac{\pi}{3}}$

Given that $\tan\frac{x}{2} = \frac{1-\cos x}{\sin x}$, evaluate $\tan\frac{5\pi}{12}$.

$$\tan\frac{5\pi}{12} = \tan\frac{5\pi/6}{2} = \frac{1-\cos 5\pi/6}{\sin 5\pi/6}$$

$$= \frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{\frac{1}{2}} = \left(1+\frac{\sqrt{3}}{2}\right)\left(\frac{2}{1}\right) = \boxed{2+\sqrt{3}}$$

Prove.

$\sin 3x = 3\sin x - 4\sin^3 x$

$$\begin{aligned}
LHS &= \sin(2x+x) \\
&= \sin 2x\cos x + \cos 2x \sin x \\
&= 2\sin x\cos x \cdot \cos x + (1-2\sin^2 x)\sin x \\
&= 2\sin x\cos^2 x + \sin x - 2\sin^3 x \\
&= 2\sin x(1-\sin^2 x) + \sin x - 2\sin^3 x \\
&= 2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x \\
&= 3\sin x - 4\sin^3 x \\
&= RHS
\end{aligned}$$