Homework #11 (due Monday 10/27)

- Final Exam Practice Problems

Review Session - 3:45pm, Monday, 10/27

Final Exam - 1:00pm, Tues. 10/28

Find the exact value of $\cos\frac{\pi}{12}\cos\frac{\pi}{4} - \sin\frac{\pi}{12}\sin\frac{\pi}{4}$.

$$= \cos\left(\frac{\pi}{12} + \frac{\pi}{4}\right)$$

$$= \cos\left(\frac{\pi}{12} + \frac{3\pi}{12}\right)$$

$$= \cos\left(\frac{4\pi}{12}\right)$$

$$= \cos\frac{\pi}{3} = \boxed{1/2}$$

Given $\vec{v} = \langle 3, -2\rangle$, $\vec{w} = \langle -7, 1\rangle$

1. Find $\vec{v} - \vec{w}$.

$$\langle 10, -3\rangle$$

2. Find $|\vec{v}|$.

$$\sqrt{3^2 + (-2)^2} = \boxed{\sqrt{13}}$$

3. Find $\vec{v} \cdot \vec{w}$.

$$3(-7) + (-2)(1) = -21 - 2 = \boxed{-23}$$

4. Find a unit vector $\vec{u}$ in the same direction as $\vec{v}$.

$$\left\langle \frac{3}{\sqrt{13}}, \frac{-2}{\sqrt{13}} \right\rangle = \left\langle \frac{3\sqrt{13}}{13}, \frac{-2\sqrt{13}}{13} \right\rangle$$

Given the complex numbers $z_1 = 5\cos(45°) + 5i\sin(45°)$ and $z_2 = 10\cos(50°) + 10i\sin(50°)$,

$5 \text{ cis } 45°$ $10 \text{ cis } 50°$

5. Find $z_1 z_2$.

$(5)(10) \text{ cis}(45° + 50°) = \boxed{50 \text{ cis } 95°}$

6. Find $\frac{z_2}{z_1}$.

$\frac{10}{5} \text{ cis}(50° - 45°) = \boxed{2 \text{ cis } 5°}$

7. Convert z_1 into standard form. $z = a + bi$

$z_1 = 5\cos 45° + 5i\sin 45° = \boxed{\frac{5}{\sqrt{2}} + \frac{5}{\sqrt{2}} i}$

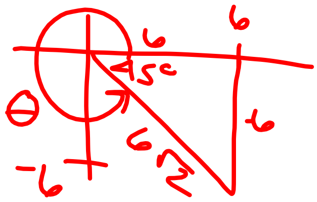
$= \frac{5\sqrt{2}}{2} + \frac{5\sqrt{2}}{2} i$

Given the complex number $z = 6 - 6i$,

8. Find the modulus $|z|$.

$\sqrt{6^2 + (-6)^2} = \boxed{6\sqrt{2}}$

9. Find the argument θ of z.



$\boxed{315°}$

Review:

10. Find the length of an arc that subtends an angle of 120° on a circle whose diameter is 12 cm.

$s = ?$ $\theta = 120°$ $r = 6\text{cm}$

$s = r\theta = 6\text{cm} \cdot \frac{120°}{1} \cdot \frac{\pi}{180°} = \boxed{4\pi \text{ cm}}$

Evaluate the following trigonometric expressions:

11. $\sec\frac{\pi}{4}$ $\sqrt{2}$

12. $\cos(-420°)$ $1/2$

13. $\tan(-90°)$ undefined

14. $\sin 135°$ $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

Evaluate the inverse trig function. Give your answer in radians.

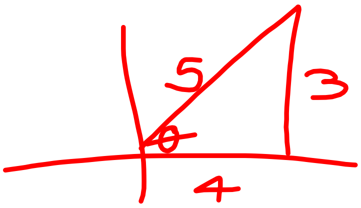
15. $\sin^{-1}\left(-\frac{1}{2}\right)$ $-\frac{\pi}{6}$

16. $\cos^{-1}\frac{1}{\sqrt{2}}$ $\frac{\pi}{4}$

17. Simplify. $\cos 125° \cos 55° - \sin 125° \sin 55°$

$= \cos(125° + 55°) = \cos 180° = \boxed{-1}$

18. Evaluate. $\sin\left[\cos^{-1}\left(\frac{4}{5}\right)\right] = \boxed{\frac{3}{5}}$



$f(x) = -3\csc\left(4x + \frac{\pi}{2}\right) - 2$

a. "amplitude" 3

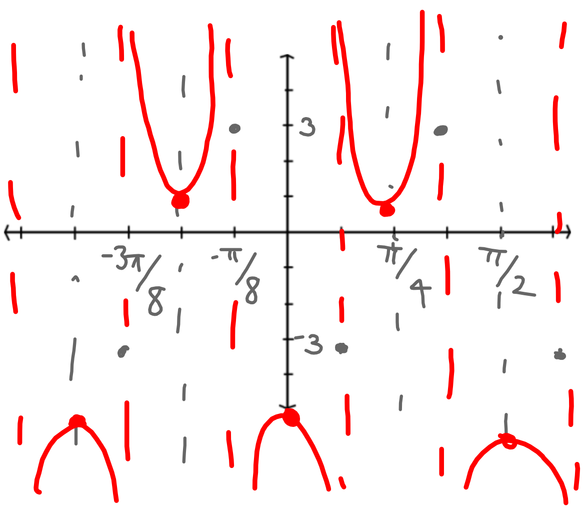
b. period $\frac{2\pi}{4} = \frac{\pi}{2}$

c. horizontal shift

$\frac{\pi/2}{4} = -\pi/8$ (left)

d. vertical shift

-2 (down)



$f(x) = \frac{2}{3}\cot\left(3x - \frac{3\pi}{4}\right) + 1$

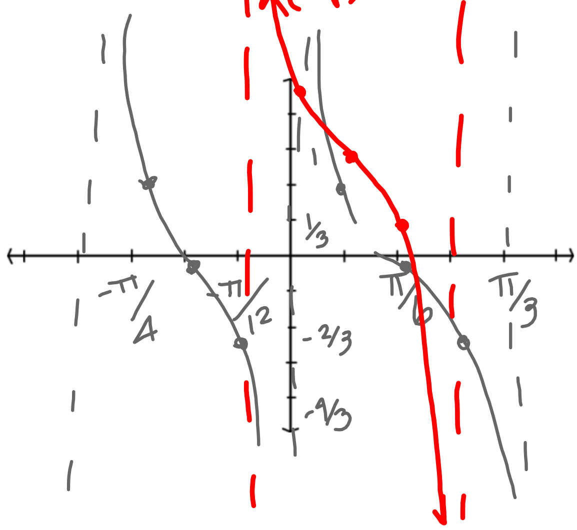
e. "amplitude" $2/3$

f. period $\pi/3$

g. horizontal shift $\frac{3\pi/4}{3} = \frac{\pi}{4}$ (right)

h. vertical shift

1 (up)

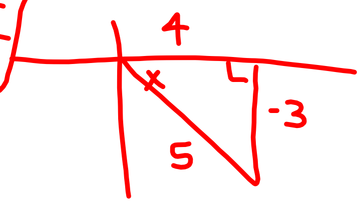


Angle	Quadrant (I, II, III, or IV)	Reference angle (in degrees)
$-240°$	II	$60°$
$310°$	IV	$50°$
$\frac{7\pi}{6}$	III	$30°$
$\frac{9\pi}{4}$	I	$45°$
$\frac{2\pi}{5}$	I	$72°$

$$\frac{2\cancel{\pi}}{\cancel{5}} \cdot \frac{\overset{36}{\cancel{180}}°}{\cancel{\pi}}$$

Given that $\sin x = \frac{-3}{5}$ and x is in quadrant IV, find sin 2x, cos 2x, and tan 2x.

$$\sin 2x = 2\sin x\cos x = 2\left(\frac{-3}{5}\right)\left(\frac{4}{5}\right) = \boxed{\frac{-24}{25}}$$



$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= \left(\frac{4}{5}\right)^2 - \left(\frac{-3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \boxed{\frac{7}{25}}$$

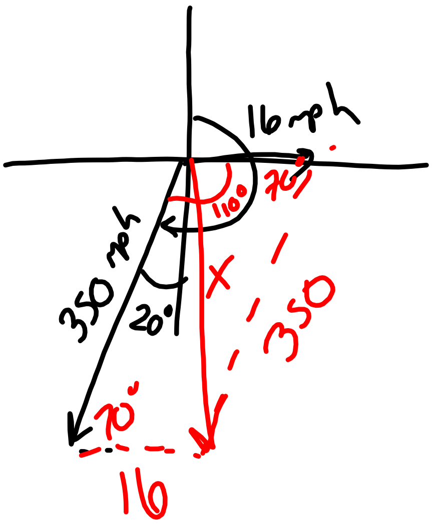
$$\tan 2x = \frac{\sin 2x}{\cos 2x} = \frac{-24/25}{7/25} = \boxed{\frac{-24}{7}}$$

$2x \in$ Q III

22. airspeed 350 mph @ heading of 200°

wind blowing from west @ 16 mph

ground speed of plane?

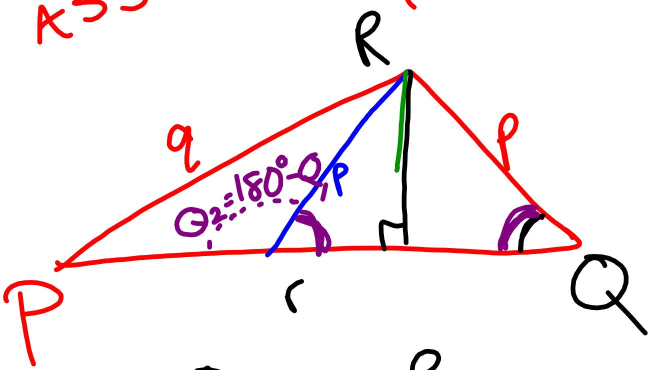


$x = \sqrt{16^2 + 350^2 - 2(16)(350)\cos 110}$

$\approx$

ASS

$P = \quad ^\circ$; $p =$; $q =$



$$\frac{\sin Q}{q} = \frac{\sin P}{p}$$

$$\sin Q = \underbrace{\frac{q \sin P}{p}}_{>1}$$

$\Rightarrow$ no sol'n

case 1:

Q = acute ∠

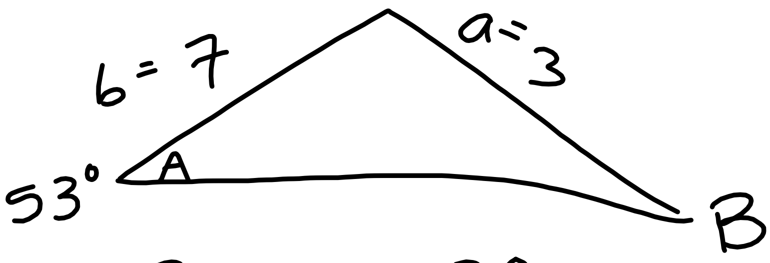
$R = 180^\circ - P - Q$

$$\frac{r}{\sin R} = \frac{p}{\sin P}$$

case 2:

$Q_2 = 180^\circ - Q_1$

$A = 53° \; ; \; a = 3 \; ; \; b = 7$



$$\frac{\sin B}{7} = \frac{\sin 53°}{3}$$

$$\sin B = \frac{7 \cdot \sin 53°}{3}$$

$$B = \sin^{-1}\left(\frac{7 \sin 53°}{3}\right) = \text{error} \Rightarrow \text{no triangle}$$