<u>6.6</u>

Solving Trigonometric Equations

$$\sin^{-1}\left(\frac{1}{2}\right) \quad \text{versus} \quad \sin x = \frac{1}{2}$$

$$= \frac{\pi}{6}$$

only one
answer
(because we restricted
the domain of sinx
to define sin⁻¹(x))

$$x = \frac{\pi}{6} + 2\pi k$$

$$x = \frac{5\pi}{6} + 2\pi k$$

$k \in \mathbb{Z}$

$\mathbb{Z}$ = the set of Integers

5π/6   π/6

Solve for $x \in [0, 2\pi)$.

2. $2 \sin x = \sqrt{3}$

$$\sin x = \frac{\sqrt{3}}{2}$$

$$\boxed{x = \frac{\pi}{3}, \frac{2\pi}{3}}$$
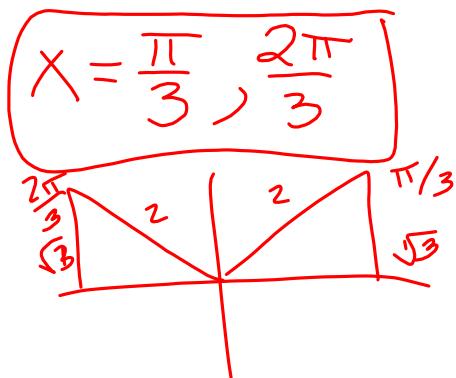
4. $\cos x - 1 = 0$

$$\cos x = 1$$

$$\boxed{x = 0}$$

6. $2 \sin x \cos x = \sqrt{3} \sin x$

$$2 \sin x \cos x - \sqrt{3} \sin x = 0$$

$$\sin x \left(2 \cos x - \sqrt{3}\right) = 0$$

$$\underline{\sin x = 0} \quad ; \quad 2 \cos x - \sqrt{3} = 0$$

$$\boxed{x = 0, \pi} \qquad 2 \cos x = \sqrt{3}$$

$$\cos x = \frac{\sqrt{3}}{2}$$

$$\boxed{x = \frac{\pi}{6}, \frac{11\pi}{6}}$$

$$x^2 = x$$
$$x = 1$$

$$x^2 - x = 0$$
$$x(x-1) = 0$$
$$x = 0, \quad x = 1$$

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Algebra Review

$(x - 2)(x - 3)(x - 4) = 0$

$x - 2 = 0$, $x - 3 = 0$, $x - 4 = 0$

$x = 2$ $x = 3$ $x = 4$

The **Zero Product Property** states:

If $AB = 0$, then $A = 0$ or $B = 0$.

$x^2 = 9$

$x = \pm 3$

The **Square Root Theorem** states:

If $[f(x)]^2 = c$, then $f(x) = \pm\sqrt{c}$

6.6 Solving Trigonometric Equations
Solve for all values of x in the interval [0, 2pi)

8. $\cos^2 x - 1 = 0 \longrightarrow (\cos x - 1)(\cos x + 1) = 0$

$\cos^2 x = 1$

$\cos x = \pm 1$

$\cos x - 1 = 0, \quad \cos x + 1 = 0$

$\cos x = 1, \quad \cos x = -1$

$\boxed{x = 0, \pi}$

$\longrightarrow -\sin^2 x = 0$

$\sin^2 x = 0$

$\sin x = 0$

10. $\sec^2 x + \sqrt{3} \sec x - \sqrt{2} \sec x - \sqrt{6} = 0$

$\sec x (\sec x + \sqrt{3}) - \sqrt{2}(\sec x + \sqrt{3}) = 0$

$(\sec x + \sqrt{3})(\sec x - \sqrt{2}) = 0$

$\sec x + \sqrt{3} = 0 \quad ; \quad \sec x - \sqrt{2} = 0$

$\sec x = -\sqrt{3} \qquad\qquad \sec x = \sqrt{2}$

$\boxed{x = \sec^{-1}(\sqrt{3})}$

$\boxed{\& x = 2\pi - \sec^{-1}(\sqrt{3})}$

$\boxed{x = \dfrac{\pi}{4}, \dfrac{7\pi}{4}}$
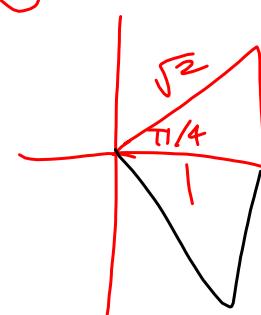
$\underline{\sec^{-1}(x) = \cos^{-1}\left(\dfrac{1}{x}\right)}$

14. $2\cos^2 x + 1 = -3\cos x$

$2\cos^2 x + 3\cos x + 1 = 0$

Let $u = \cos x$

$2u^2 + 3u + 1 = 0$

$(2u + 1)(u + 1) = 0$

$(2\cos x + 1)(\cos x + 1) = 0$

$2\cos x + 1 = 0$, $\cos x + 1 = 0$

$2\cos x = -1$ $\cos x = -1$

$\cos x = -\frac{1}{2}$

$\boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3}}$ $\boxed{x = \pi}$

$\frac{2\pi}{3}$ $\frac{3\pi}{3} = \pi$ $\frac{\pi}{3}$ $\frac{\pi}{3}$ $\frac{4\pi}{3}$
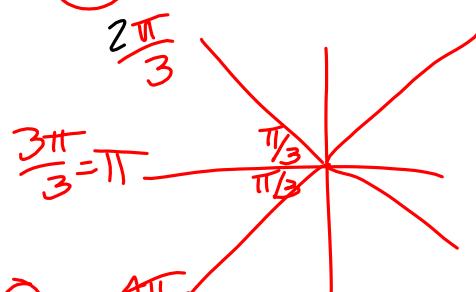
18. $4\cos^3 x = 3\cos x$

$4\cos^3 x - 3\cos x = 0$

$\cos x (4\cos^2 x - 3) = 0$

$\cos x = 0$ $4\cos^2 x - 3 = 0$

$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}}$ $4\cos^2 x = 3$

$\cos^2 x = \frac{3}{4}$

$\cos x = \frac{\pm\sqrt{3}}{2}$

$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$