6.6  #1-21 odd  11        finding solutions between 0 and 2pi        } due Tues 01/31
    #61-69 odd  5        finding all possible solutions (+2pi*k)
    #71-83 odd  7

Quiz on solving equations next week; Test #4 - Feb 6-8ish?

Solve for $x \in [0, 2\pi)$

20. $\tan^2 x + \tan x - \sqrt{3} = \sqrt{3} \tan x$

$$\tan^2 x + \tan x - \sqrt{3}\tan x - \sqrt{3} = 0$$
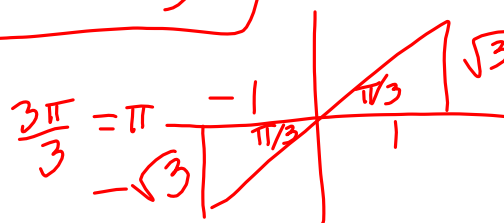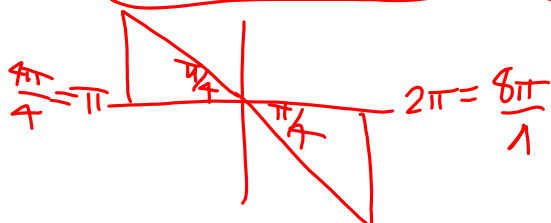
$$\tan x(\tan x + 1) - \sqrt{3}(\tan x + 1) = 0$$

$$(\tan x + 1)(\tan x - \sqrt{3}) = 0$$

$$\tan x = -1 \qquad\qquad \tan x = \sqrt{3}$$

$$\boxed{x = \frac{3\pi}{4}, \frac{7\pi}{4}} \qquad \boxed{x = \frac{\pi}{3}, \frac{4\pi}{3}}$$

$\frac{4\pi}{4} = \pi$    $2\pi = \frac{8\pi}{4}$     $\frac{3\pi}{3} = \pi$

$x \in [0, 2\pi)$

22. $\cos^4 x = \cos^2 x$

$\cos^4 x - \cos^2 x = 0$

$\cos^2 x (\cos^2 x - 1) = 0$

$\cos^2 x = 0 \qquad\qquad \cos^2 x = 1$

$\cos x = 0 \qquad\qquad \cos x = \pm 1$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}, 0, \pi$$

**New Directions:** <u>Find ALL the solutions</u> (not just in $[0, 2\pi)$ )

62. $\sec 3x - \frac{2\sqrt{3}}{3} = 0$
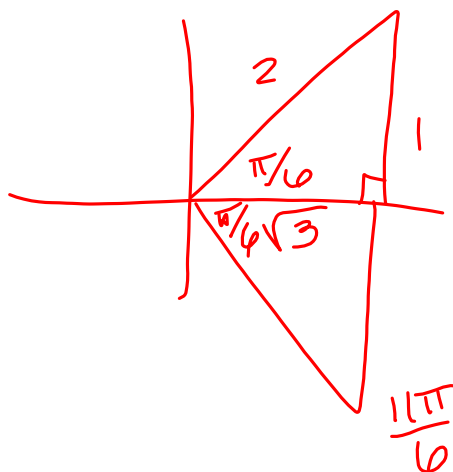
$\sec 3x = \frac{2\sqrt{3}}{3}$

$\sec \boxed{3x} = \frac{2}{\sqrt{3}}$

$3x = \frac{\pi}{6} + 2\pi k$
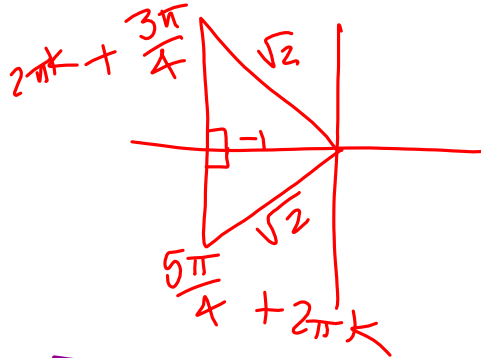
$$x = \frac{\pi}{18} + \frac{2\pi k}{3}$$

$3x = \frac{11\pi}{6} + 2\pi k$

$$x = \frac{11\pi}{18} + \frac{2\pi k}{3}$$



$2$

$1$

$\pi/6$

$\sqrt{3}$

$\frac{11\pi}{6}$

68. $\cos\left(2x - \dfrac{\pi}{4}\right) = -\dfrac{\sqrt{2}}{2}$

$\cos \theta = -\dfrac{1}{\sqrt{2}}$

$2\pi k + \dfrac{3\pi}{4}$    $\sqrt{2}$

$-1$

$\dfrac{5\pi}{4} + 2\pi k$   $\sqrt{2}$

$2x - \dfrac{\pi}{4} = \dfrac{3\pi}{4} + 2\pi k$ ;    $2x - \dfrac{\pi}{4} = \dfrac{5\pi}{4} + 2\pi k$

$2x = \pi + 2\pi k$

$\boxed{x = \dfrac{\pi}{2} + \pi k}$

$2x = \dfrac{3\pi}{2} + 2\pi k$

$\boxed{x = \dfrac{3\pi}{4} + \pi k}$

---

Solve for $x \in [0, 2\pi)$.

$(\sin x - \cos x)^2 = 1^2$

**✱ squaring both sides may introduce extraneous solutions!**

$\sin^2 x - 2\sin x\cos x + \cos^2 x = 1$

$\underbrace{\sin^2 x + \cos^2 x}_{=1} - 2\sin x\cos x = 1$

$1 - 2\sin x\cos x = 1$
$\phantom{1-}\underbrace{2\sin x\cos x}_{\sin 2x}$

$1 - \sin 2x = 1$  ↪

$0 = \sin 2x$

$2x = 0, \pi, 2\pi, 3\pi$

$x = \cancel{0}, \dfrac{\pi}{2}, \cancel{\pi}, \dfrac{3\pi}{2}$ ?

$2\sin x\cos x = 0$
$2\sin x = 0 \qquad \cos x = 0$
$\sin x = 0 \qquad \boxed{x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}}$
$\boxed{x = 0, \pi}$

won x    $0 \le x < 2\pi$
$\qquad\quad 0 \le 2x < 4\pi$

$\pi$
$(-1, 0)$    $(1, 0)$   $0$
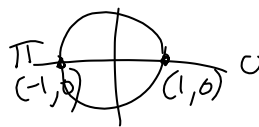
Is $\sin x - \cos x = 1$?

$\sin 0 - \cos 0 = 0 - 1 = -1 \ne 1$

$\sin \dfrac{\pi}{2} - \cos \dfrac{\pi}{2} = 1 - 0 = 1 \checkmark$

$\sin \pi - \cos \pi = 0 - (-1) = 1 \checkmark$

$\sin \dfrac{3\pi}{2} - \cos \dfrac{3\pi}{2} = -1 - 0 = -1 \ne 1$

$x \in [0, 2\pi)$

$$\cos(4x) = \frac{1}{\sqrt{2}}$$

$0 \leq x < 2\pi$

$0 \leq 4x < 8\pi$

$$4x = \frac{\pi}{4}, \frac{7\pi}{4} ; \frac{9\pi}{4}, \frac{15\pi}{4} ; \frac{17\pi}{4}, \frac{23\pi}{4} ; \frac{25\pi}{4}, \frac{31\pi}{4}$$

$$x = \frac{\pi}{16}, \frac{7\pi}{16}, \frac{9\pi}{16}, \frac{15\pi}{16}, \frac{17\pi}{16}, \frac{23\pi}{16}, \frac{25\pi}{16}, \frac{31\pi}{16}$$

$x \in [0, 2\pi)$

$$\tan(5x) = 0$$

$$5x = 0, \pi ; 2\pi, 3\pi ; 4\pi, 5\pi ; 6\pi, 7\pi ; 8\pi, 9\pi$$

$$x = 0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}, \pi, \frac{6\pi}{5}, \frac{7\pi}{5}, \frac{8\pi}{5}, \frac{9\pi}{5}$$

**4**