Quiz on solving equations Fri. Feb 3; Test #4 - Wed. Feb 8

Due Tues. 2/7:

- 7.1 #7-21 odd — solving triangles with Law of Sines
- 7.2 #9-19 odd — solving triangles with Law of Cosines
- 7.2 #25-29odd — area

Review:

10. Find the length of an arc that subtends an angle of 120° on a circle whose diameter is 12 cm.

$s = ?$ $\theta = 120^\circ$; $r = 6\text{cm}$

$$s = r\theta = \frac{\cancel{6}\text{cm} \cdot \cancel{120^\circ}^{4}}{1} \cdot \frac{\pi}{\cancel{180^\circ}_{3}} = \boxed{4\pi \text{ cm}}$$

The Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

or

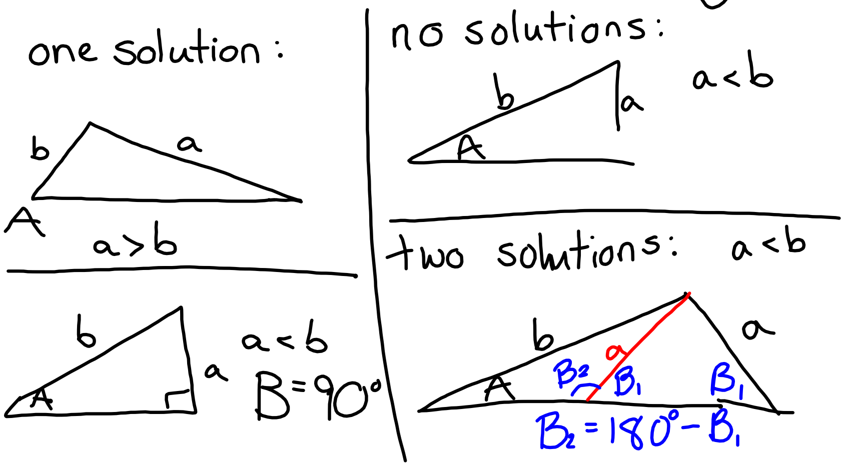
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

ASS

AAS

ASA

ASS, The Problematic Triangle

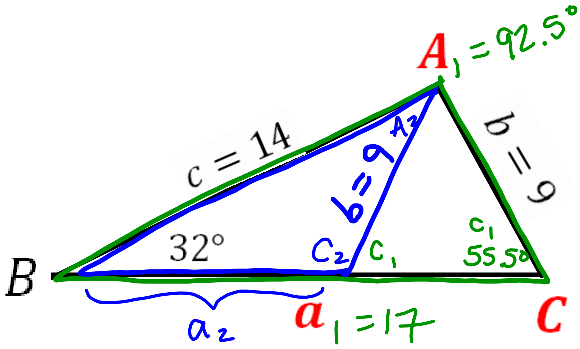


7.1 The Law of Sines, continued

ASS – Problematic Triangle

14. $B = 32°$, $c = 14$, $b = 9$

Case 1: $C \approx 55.5°$, $A \approx 92.5°$, $a \approx 17$



Case 2

$C_2 = 180° - 55.5°$

$\boxed{C_2 = 124.5°}$

$A_2 = 180° - 124.5° - 32°$

$\boxed{A_2 = 23.5°}$

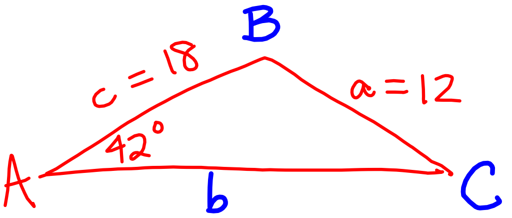
$\angle C_2 = 180° - C_1$

$$\frac{a_2}{\sin 23.5°} = \frac{9}{\sin 32°}$$

$$a_2 = \frac{9 \sin 23.5°}{\sin 32°}$$

$$\boxed{a_2 \approx 6.8}$$

16. $A = 42°, a = 12, c = 18$



ASS, $a < c$

$$\frac{\sin C}{18} = \frac{\sin 42°}{12}$$

$$\sin C = \frac{18 \sin 42°}{12}$$

$$\sin^{-1}(\sin C) = \sin^{-1}\left(\frac{18 \sin 42°}{12}\right)$$

$$C = \sin^{-1}\underbrace{\left(\frac{18 \sin 42°}{12}\right)}_{>1}$$

undefined

$\Rightarrow$ $\boxed{\text{no triangle}}$

<u>The Law of Cosines</u>

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cdot \cos B$$

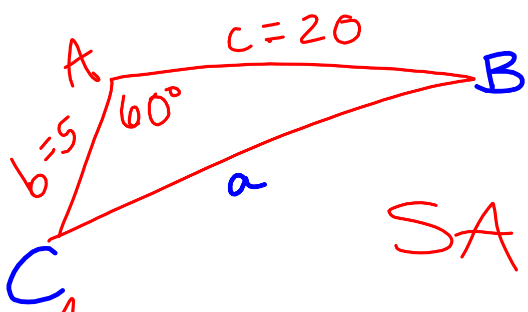
$$c^2 = a^2 + b^2 - 2ab \cdot \cos C$$

SSS

SAS

A=60°, b=5, c=20

Find side a.



SAS

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$a = \sqrt{b^2 + c^2 - 2bc \cdot \cos A}$$

$$= \sqrt{5^2 + 20^2 - 2(5)(20)\cos 60^\circ} \approx \boxed{18}$$

16. $a = 60, b = 88, c = 120.\ B = ?$

SSS

$$b^2 = a^2 + c^2 - 2ac \cdot \cos B$$

$$b^2 - b^2 + 2ac \cdot \cos B = a^2 + c^2 - 2ac\cos B + 2ac\cos B - b^2$$

$$2ac \cdot \cos B = a^2 + c^2 - b^2$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos^{-1}(\cos B) = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right)$$

$$B = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right)$$

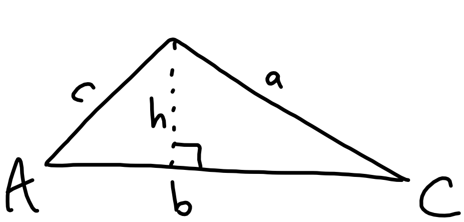
60
88
120

$$B = \cos^{-1}\left(\frac{60^2 + 120^2 - 88^2}{2(60)(120)}\right)$$

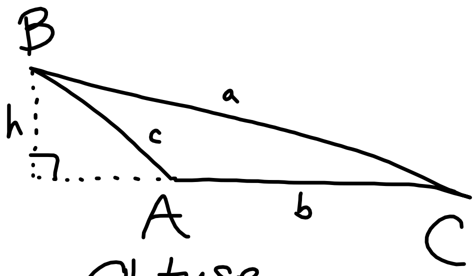
$$\approx \boxed{44.6^\circ}$$

cos⁻¹((60^2+120^2-88^2)/(2*60*120))

7.1/7.2 Area of a Triangle



Acute



Obtuse

$\sin A = \frac{h}{c} \Rightarrow c\sin A = h$

$\sin C = \frac{h}{a} \Rightarrow a\sin C = h$

$\text{area} = \frac{1}{2}\text{base} \cdot \text{height}$

$= \frac{1}{2} \cdot b \cdot c \sin A$

$= \frac{1}{2} ba \cdot \sin C$

$= \frac{1}{2} ac \sin B$

SAS cases

$\sin C = \frac{h}{a} \Rightarrow h = a\sin C$

$\sin A = \frac{h}{c} \Rightarrow h = c\sin A$

$\text{area} = \frac{1}{2} bc\sin A$

Find the area of the triangle.

$A = 50°, b = 13\ cm, c = 6\ cm$

$\text{area} = \frac{1}{2} bc \sin A$

$= \frac{1}{2}(13)(6)\sin 50°$

$\approx \boxed{29.9\ cm^2}$