Assignments for the Week of Sept. 19

- Read 6.1-6.2
- 45 minutes of Khan Academy (by Friday as usual)
- Textbook assignment due Wednesday, Sept. 21:
  6.2 #1-41 odd

Assignments for the break:

- Read 6.3
- **memorize your identities!!!**

After the break:

- 6.1 #1-69 odd (proofs)
- 6.3 #1-24 all; 30-36 all; 49-93 odd

Chapter 6 - Trigonometric Identities and Equations

Reciprocal Identities

$\csc x = \frac{1}{\sin x}$, $\sin x = \frac{1}{\csc x}$

$\sec x = \frac{1}{\cos x}$, $\cos x = \frac{1}{\sec x}$

$\cot x = \frac{1}{\tan x}$, $\tan x = \frac{1}{\cot x}$

Ratio Identities

$\tan x = \frac{\sin x}{\cos x}$, $\cot x = \frac{\cos x}{\sin x}$

Cofunction Identities

$\sin\left(\frac{\pi}{2} - x\right) = \cos x$, $\cos\left(\frac{\pi}{2} - x\right) = \sin x$

$\tan\left(\frac{\pi}{2} - x\right) = \cot x$, $\cot\left(\frac{\pi}{2} - x\right) = \tan x$

$\csc\left(\frac{\pi}{2} - x\right) = \sec x$, $\sec\left(\frac{\pi}{2} - x\right) = \csc x$

Odd-Even Identities

$\cos(-x) = \cos x$, $\sin(-x) = -\sin x$, $\tan(-x) = -\tan x$

$\sec(-x) = \sec x$, $\csc(-x) = -\csc x$, $\cot(-x) = -\cot x$

Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$

$1 + \cot^2 x = \csc^2 x$

$\tan^2 x + 1 = \sec^2 x$

To <u>prove an identity</u> means to show that one side of the identity can be rewritten in a form that is identical to the other side.

There is no one method that works for every identity, the following are some helpful guidelines:

- remember that you are trying to prove that this equation is true, so you can't treat it like an equation -- no working on both sides (e.g. you can't add something to both sides, or divide both sides by something). You must start with one side and rewrite it until it is equal to the other side. It is okay to meet in the middle if you get stuck and must work from both sides.
- if one side is more complicated than the other, start with the more complicated side and try to simplify it
- use rules of algebra to find common denominators, add fractions, square binomials, factor, multiply by a form of 1, add 0, etc.
- apply known identities to rewrite parts of an expression in a more useful form, e.g. since $\sin^2 x + \cos^2 x = 1$, you can replace the expression $\sin^2 x + \cos^2 x$ with 1.
- when in doubt, rewrite in terms of sine and cosine

Useful formulas from Algebra:

$$a^2 - b^2 = (a-b)(a+b)$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$(a+b)^2 \neq a^2 + b^2$

$(a+b)(a+b)$

6.2 - Sum and Difference Identities

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$

$$\sin(a-b) = \sin a \cos b - \cos a \sin b$$

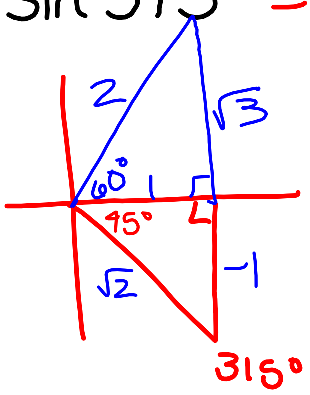
$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

$$\sin 375^\circ = \sin(315^\circ + 60^\circ)$$



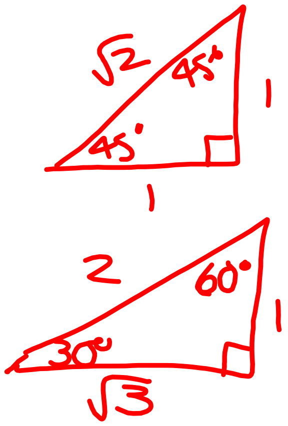
$$= \sin 315^\circ \cos 60^\circ + \cos 315^\circ \sin 60^\circ$$

$$= \left(\frac{-1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \left(\frac{-\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{-\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$$

$$\cos\left(\frac{\pi}{4}-\frac{\pi}{3}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{3}+\sin\frac{\pi}{4}\sin\frac{\pi}{3}$$



$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)+\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4}$$

$$= \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$$

$$\sin 167^\circ \cos 107^\circ - \cos 167^\circ \sin 107^\circ$$

$$\sin a \cos b - \cos a \sin b = \sin(a-b)$$

$$= \sin(167^\circ - 107^\circ)$$

$$= \sin 60^\circ$$

$$= \boxed{\sqrt{3}/2}$$

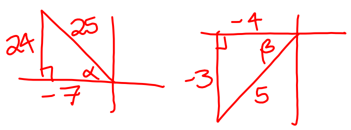
$\sin x \cos 3x + \cos x \sin 3x$

$= \sin(x + 3x)$

$= \sin 4x$

Given $\sin\alpha = \frac{24}{25}$, $\alpha \in Q\,\text{II}$

$\cos\beta = \frac{-4}{5}$, $\beta \in Q\,\text{III}$



Find $\sin(\alpha-\beta)$, $\cos(\alpha-\beta)$, $\tan(\alpha-\beta)$ & determine the quadrant in which $\alpha-\beta$ lies.

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= \left(\frac{24}{25}\right)\left(\frac{-4}{5}\right) - \left(\frac{-7}{25}\right)\left(\frac{-3}{5}\right) = \frac{-96}{125} - \frac{21}{25}$

$\boxed{\sin(\alpha-\beta) = -\frac{117}{125}}$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$= \left(\frac{-7}{25}\right)\left(\frac{-4}{5}\right) + \left(\frac{24}{25}\right)\left(\frac{-3}{5}\right) = \frac{28}{125} - \frac{72}{125}$

$\boxed{\cos(\alpha-\beta) = -\frac{44}{125}}$

$\tan(\alpha-\beta) = \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)} = \frac{\left(\frac{-117}{125}\right)}{\left(\frac{-44}{125}\right)} = \frac{117}{44}$

$\boxed{\tan(\alpha-\beta) = \frac{117}{44}}$

$\boxed{\alpha-\beta \text{ is in Quadrant III}}$

$(\sin, \cos < 0,\ \tan > 0)$

Given $\cos\alpha = \frac{8}{17}$, $\alpha \in Q\,IV$

$\sin\beta = \frac{-24}{25}$, $\beta \in Q\,III$

find $\sin(\alpha+\beta)$, $\cos(\alpha+\beta)$, $\tan(\alpha+\beta)$, & determine the quadrant in which $\alpha+\beta$ lies.

*Pythagorean triples that are useful to know:
3, 4, 5 ; 5, 12, 13 ; 7, 24, 25 ;
& 8, 15, 17

Cofunction Identities

The function of an angle is equal to the cofunction of its complement.

θ & $90° - \theta$ or θ & $\frac{\pi}{2} - \theta$
are complementary angles

$$\cos\left(\frac{\pi}{2} - x\right) = \cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x$$

$$= 0 \cdot \cos x + 1 \cdot \sin x$$

$$= \boxed{\sin x}$$

Double-Angle Identities

$\sin 2\theta = \sin(\theta + \theta)$

$= \sin\theta\cos\theta + \cos\theta\sin\theta$

$\boxed{\sin 2\theta = 2\sin\theta\cos\theta}$

"the sine of twice an angle is equal to 2 times the sine of the angle times the cosine of the angle"

$\sin(42x) = \sin[2(21x)] = 2\sin 21x\cos 21x$

$\sin(8x) = \sin[2(4x)] = 2\underbrace{\sin 4x}_{\sin 2(2x)}\underbrace{\cos 4x}_{\cos 2(2x)}$

$\boxed{\sin 2\theta = 2\sin\theta\cos\theta}$

The sine of twice any angle is equal to two times the sine of that angle times the cosine of that angle.

$\sin 6\theta = 2\sin 3\theta\cos 3\theta$

$\sin 8\theta = 2\sin 4\theta\cos 4\theta$

$\sin 44\theta = 2\sin 22\theta\cos 22\theta$

$\sin 3\theta = 2\sin\frac{3\theta}{2}\cos\frac{3\theta}{2}$

$= \sin(2\theta + \theta)$

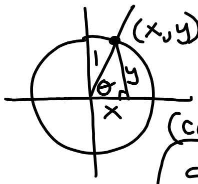
even multiple $\Rightarrow$ double $\angle$'s

odd multiple $\Rightarrow$ sum

$$\cos 2\theta = \cos(\theta+\theta)$$
$$= \cos\theta\cos\theta - \sin\theta\sin\theta$$
$$= (\cos\theta)^2 - (\sin\theta)^2$$
$$\boxed{\cos 2\theta = \cos^2\theta - \sin^2\theta}$$

$\sin\theta^2 = \sin(\theta^2)$

$\cancel{}$ $\sin^2\theta$



$(x, y) = (\cos\theta, \sin\theta)$

$x^2 + y^2 = 1$

$(\cos\theta)^2 + (\sin\theta)^2 = 1^2$

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

Pythagorean Identity

$$\sin^2\theta = 1 - \cos^2\theta$$
$$\cos^2\theta = 1 - \sin^2\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= \cos^2\theta - (1 - \cos^2\theta)$$
$$\boxed{\cos 2\theta = 2\cos^2\theta - 1}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= (1 - \sin^2\theta) - \sin^2\theta$$
$$\boxed{\cos 2\theta = 1 - 2\sin^2\theta}$$

$$\tan 2\theta = \tan(\theta+\theta)$$
$$= \frac{\tan\theta + \tan\theta}{1 - \tan\theta\tan\theta}$$
$$\boxed{\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}}$$

Double-Angle Identities

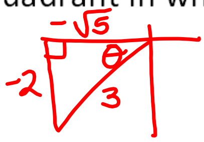
$$\sin 2x = 2\sin x \cos x$$

$$\begin{aligned}\cos 2x &= \cos^2 x - \sin^2 x \\ &= 2\cos^2 x - 1 \\ &= 1 - 2\sin^2 x\end{aligned}$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

Given $\sin\theta = -\frac{2}{3}$, $\theta \in QIII$,

Find $\sin 2\theta$, $\cos 2\theta$, $\tan 2\theta$, and the quadrant in which 2θ lies.



$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{-2}{3}\right)\left(\frac{-\sqrt{5}}{3}\right)$

$\boxed{\sin 2\theta = \frac{4\sqrt{5}}{9}}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(\frac{-\sqrt{5}}{3}\right)^2 - \left(\frac{-2}{3}\right)^2$

$= \frac{5}{9} - \frac{4}{9}$

$\boxed{\cos 2\theta = 1/9}$

$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{\frac{4\sqrt{5}}{9}}{1/9} = \boxed{4\sqrt{5} = \tan 2\theta}$

$\boxed{2\theta \in QI}$ (sin, cos, tan > 0)