Assignments for the Week of Sept. 19

- Read 6.1-6.2
- 45 minutes of Khan Academy (by Friday as usual)
- Textbook assignment due Wednesday, Sept. 21:
  6.2 #1-41 odd

Assignment for the break:
memorize your identities!!!

Coming up after the break:

- 6.1 #1-69 odd (proofs)
- 6.3 #1-24 all; 30-36 all; 49-93 odd

$$\frac{1+\frac{\sqrt{3}}{3}}{1-\frac{\sqrt{3}}{3}} = \frac{\frac{3}{3}+\frac{\sqrt{3}}{3}}{\frac{3}{3}-\frac{\sqrt{3}}{3}} = \frac{\frac{3+\sqrt{3}}{3}}{\frac{3-\sqrt{3}}{3}}$$

$$= \frac{3+\sqrt{3}}{\cancel{3}} \cdot \frac{\cancel{3}}{3-\sqrt{3}} = \frac{3+\sqrt{3}}{3-\sqrt{3}} \cdot \frac{3+\sqrt{3}}{3+\sqrt{3}}$$

$$= \frac{(3+\sqrt{3})(3+\sqrt{3})}{(3-\sqrt{3})(3+\sqrt{3})} = \frac{9+6\sqrt{3}+3}{3^2-(\sqrt{3})^2} = \frac{12+6\sqrt{3}}{9-3}$$

$$= \frac{\cancel{6}(2+\sqrt{3})}{\cancel{6}} = 2+\sqrt{3}$$

Chapter 6 - Trigonometric Identities and Equations

Reciprocal Identities

$\csc x = \frac{1}{\sin x}$, $\sin x = \frac{1}{\csc x}$

$\sec x = \frac{1}{\cos x}$, $\cos x = \frac{1}{\sec x}$

$\cot x = \frac{1}{\tan x}$, $\tan x = \frac{1}{\cot x}$

Cofunction Identities

$\sin\left(\frac{\pi}{2} - x\right) = \cos x$, $\cos\left(\frac{\pi}{2} - x\right) = \sin x$

$\tan\left(\frac{\pi}{2} - x\right) = \cot x$, $\cot\left(\frac{\pi}{2} - x\right) = \tan x$

$\csc\left(\frac{\pi}{2} - x\right) = \sec x$, $\sec\left(\frac{\pi}{2} - x\right) = \csc x$

Ratio Identities

$\tan x = \frac{\sin x}{\cos x}$, $\cot x = \frac{\cos x}{\sin x}$

Odd-Even Identities

$\cos(-x) = \cos x$, $\sin(-x) = -\sin x$, $\tan(-x) = -\tan x$

$\sec(-x) = \sec x$, $\csc(-x) = -\csc x$, $\cot(-x) = -\cot x$

Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$

$1 + \cot^2 x = \csc^2 x$

$\tan^2 x + 1 = \sec^2 x$

The other two Pythagorean Identities are derived from the first.

$$\frac{\sin^2 x + \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\frac{\sin^2 x}{\cos^2 x} = \left(\frac{\sin x}{\cos x}\right)^2$$

$$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\boxed{\tan^2 x + 1 = \sec^2 x}$$

$$\frac{\sin^2 x + \cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$$

$$\boxed{1 + \cot^2 x = \csc^2 x}$$

Sum and Difference Identities

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$
$$\sin(a-b) = \sin a \cos b - \cos a \sin b$$
$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$
$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$
$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$
$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

Double-Angle Identities

$$\sin 2x = 2 \sin x \cos x$$
$$\begin{aligned} \cos 2x &= \cos^2 x - \sin^2 x \\ &= 2\cos^2 x - 1 \\ &= 1 - 2\sin^2 x \end{aligned}$$
$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

Half-Angle Identities

$\sin\frac{x}{2} = ?$

$\cos 2\theta = 1 - 2\sin^2\theta$

Let $\theta = \frac{x}{2}$.

$\cos 2\left(\frac{x}{2}\right) = 1 - 2\sin^2\left(\frac{x}{2}\right)$

$\cos x = 1 - 2\left[\sin\frac{x}{2}\right]^2$

$2\left[\sin\frac{x}{2}\right]^2 = 1 - \cos x$

$\left[\sin\frac{x}{2}\right]^2 = \frac{1-\cos x}{2}$

$\boxed{\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}}$

$\cos\frac{x}{2} = ?$

$\cos 2\theta = 2\cos^2\theta - 1$

$\cos 2\left(\frac{x}{2}\right) = 2\cos^2\frac{x}{2} - 1$

$1 + \cos x = 2\cos^2\frac{x}{2}$

$\frac{1+\cos x}{2} = \cos^2\frac{x}{2}$

$\boxed{\pm\sqrt{\frac{1+\cos x}{2}} = \cos\frac{x}{2}}$

Half-Angle Identities

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}} \quad , \quad \cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$$

$$\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$$

$$= \frac{\sin x}{1+\cos x}$$

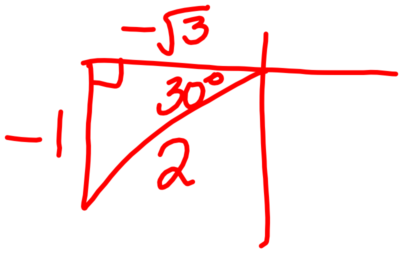
$$\boxed{\tan\frac{x}{2} = \frac{1-\cos x}{\sin x}}$$

$$\tan\frac{7\pi}{12} = \tan\frac{7\pi/6}{2} \quad 7\pi/6 = x$$

$$\tan\frac{x}{2} = \frac{1-\cos x}{\sin x}$$

$$\frac{7\pi}{12} = \frac{1}{2}\left(\frac{7\pi}{6}\right)$$

$$= \frac{1-\cos\frac{7\pi}{6}}{\sin\frac{7\pi}{6}}$$



$$= \frac{1-\left(\frac{-\sqrt{3}}{2}\right)}{\frac{-1}{2}}$$

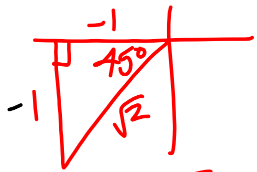
$$= \frac{\frac{2}{2}+\frac{\sqrt{3}}{2}}{\frac{-1}{2}} = \frac{2+\sqrt{3}}{\cancel{2}} \cdot \frac{-\cancel{2}^{\,-1}}{1} = \boxed{-2-\sqrt{3}} = \tan\frac{7\pi}{12}$$

6.3 Evaluate using the half-angle identity.

14. sin 112.5° $= \sin\frac{225^\circ}{2} \quad 225^\circ = x$

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$$

$$= +\sqrt{\frac{1-\cos 225^\circ}{2}}$$



$$= +\sqrt{\frac{1-\left(\frac{-\sqrt{2}}{2}\right)}{2}} \qquad \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$$

$$= +\sqrt{\frac{\frac{2}{2}+\frac{\sqrt{2}}{2}}{\frac{2}{1}}} = +\sqrt{\frac{2+\sqrt{2}}{2}\cdot\frac{1}{2}}$$

$$= +\sqrt{\frac{2+\sqrt{2}}{4}} = +\frac{\sqrt{2+\sqrt{2}}}{\sqrt{4}}$$

$$= \boxed{+\frac{\sqrt{2+\sqrt{2}}}{2}}$$

To <u>prove an identity</u> means to show that one side of the identity can be rewritten in a form that is identical to the other side.

There is no one method that works for every identity, the following are some helpful guidelines:

- remember that you are trying to prove that this equation is true, so you can't treat it like an equation -- no working on both sides (e.g. you can't add something to both sides, or divide both sides by something). You must start with one side and rewrite it until it is equal to the other side. It is okay to meet in the middle if you get stuck and must work from both sides.
- if one side is more complicated than the other, start with the more complicated side and try to simplify it
- use rules of algebra to find common denominators, add fractions, square binomials, factor, multiply by a form of 1, add 0, etc.
- apply known identities to rewrite parts of an expression in a more useful form, e.g. since $\sin^2 x + \cos^2 x = 1$, you can replace the expression $\sin^2 x + \cos^2 x$ with 1.
- when in doubt, rewrite in terms of sine and cosine

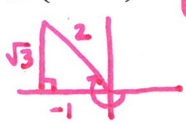
Trigonometry Test #2
Brewer

Name: KEY
16 September 2016

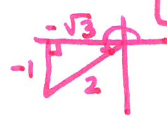
Part I - Review

Evaluate the trigonometric function for the given angle. Note that to *evaluate* does *not* mean to convert the angle from radians to degrees or vice-versa.

1. $\tan(-240°) =$ $\boxed{-\sqrt{3}}$ /4



2. $\sec\frac{7\pi}{6} =$ $\boxed{-\frac{2}{\sqrt{3}}}$ /4



Match the identities. *Print the letters neatly in the blanks provided - if I can't tell what letter you wrote, I will mark it wrong.*

__f__ 3. $\cot x =$ /4 ea

__g__ 4. $\tan x =$

__e__ 5. $\sec x =$

__c__ 6. $\csc x =$

c. $\frac{1}{\sin x}$

e. $\frac{1}{\cos x}$

f. $\frac{\cos x}{\sin x}$

g. $\frac{\sin x}{\cos x}$

Part II – Graphing Sum Functions /10 ea

Draw enough of each function to fill the entire graph (at least 2 periods of the trigonometric part). <u>You MUST include dotted line drawings of both preliminary graphs, and your final sum graph should be with a dark line or color.</u>

7. $y = 2\sin 2x + \cos x$

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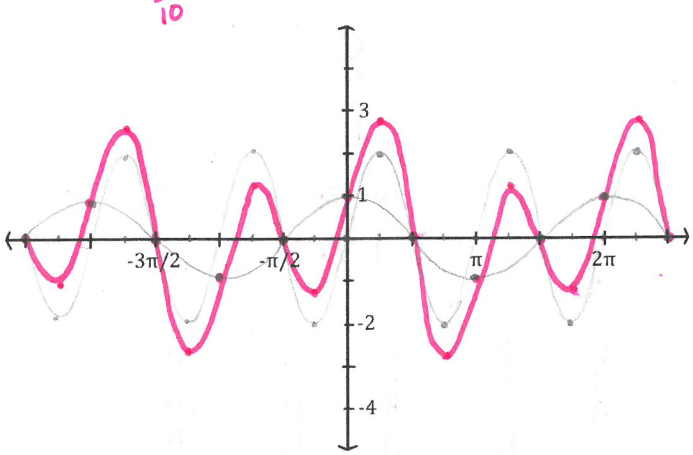
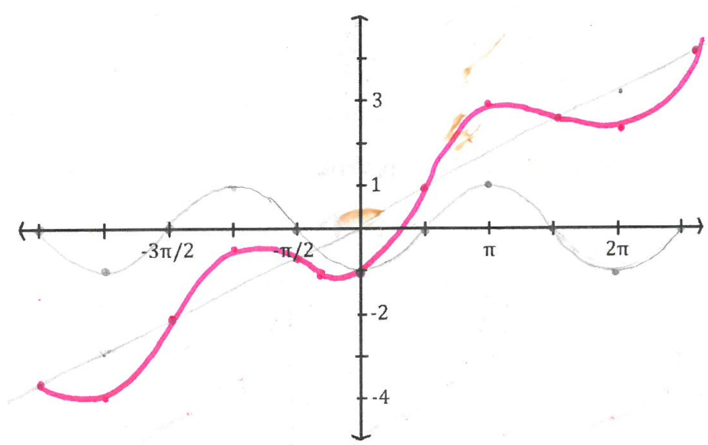


8. $y = \frac{1}{2}x - \cos x$



Part III – Graphing with Transformations

List the amplitude, period, horizontal (phase) shift, and vertical shift, and graph at least one period of each of the following functions. If a particular function does not have a horizontal or vertical shift, write "none." It is *recommended* that you draw your initial unshifted function $y = af(bx)$ with a dotted line, and your final graph with a dark line or color. **Make sure I can tell which line is your final graph, and make sure you include any asymptotes of your final graph with dotted lines.** **Don't forget to label both your x- and y-axes**.

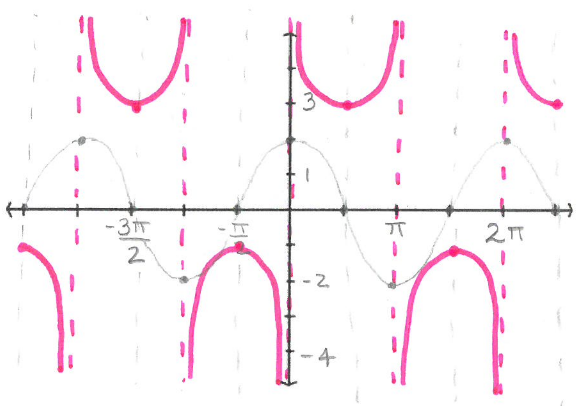
9. $y = 2\sec\left(x - \frac{\pi}{2}\right) + 1$

amplitude: 2

period: 2π

horizontal shift: $\frac{\pi}{2} \rightarrow$ (1 tick)

vertical shift: $1 \uparrow$ (1 tick)



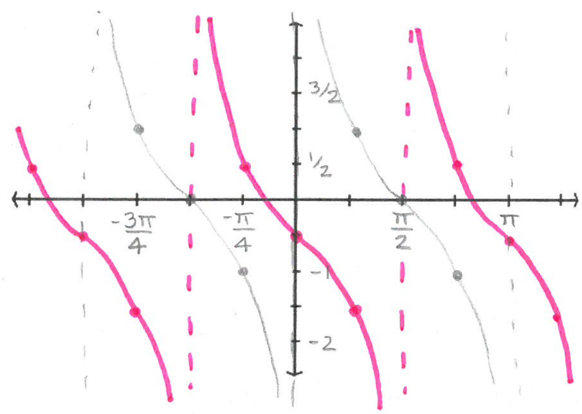
11. $y = \cot\left(x + \frac{3\pi}{2}\right) - \frac{1}{2}$

amplitude: 1

period: π

horizontal shift: $-\frac{3\pi}{2} \leftarrow$ (6 ticks)

vertical shift: $\frac{1}{2} \downarrow$ (1 tick)



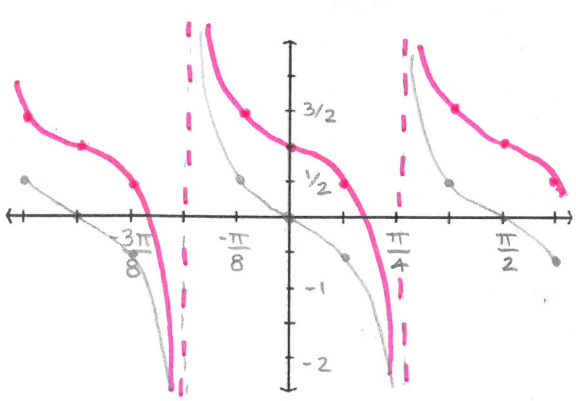
10. $y = -\frac{1}{2}\tan(2x) + 1$

amplitude: $\frac{1}{2}$

period: $\frac{\pi}{2}$

horizontal shift: none

vertical shift: $1 \uparrow$ (2 ticks)



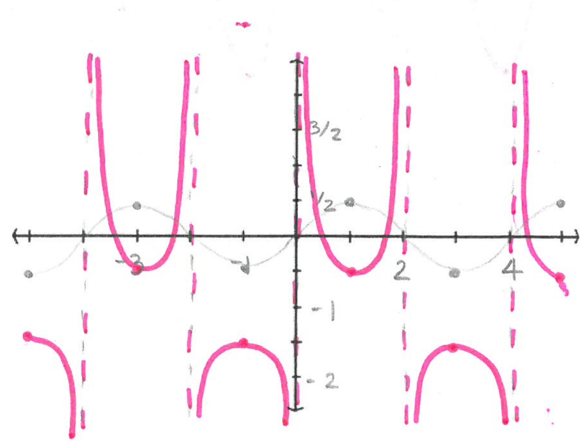
12. $y = \frac{1}{2}\csc\frac{\pi}{2}x - 1$

amplitude: $\frac{1}{2}$

period: 4

horizontal shift: none

vertical shift: $1 \downarrow$ (2 ticks)



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13. Consider the function $y = 5\cos\left(3x + \frac{\pi}{6}\right) - 1$.

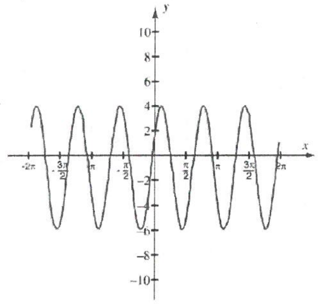
a. Find the amplitude 5

b. Find the period $2\pi/3$

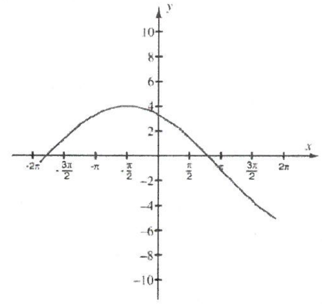
c. Find the phase shift $-\pi/18$

d. Which is the graph of the function? C

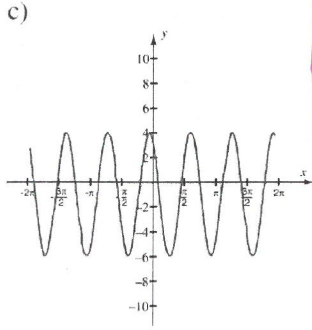
a)



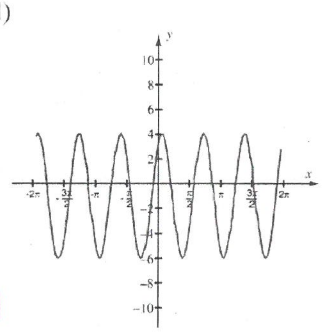
b)



c)



d)



14. Consider the function $y = 2\sin\left(\frac{x}{2} + \pi\right) + 1$.

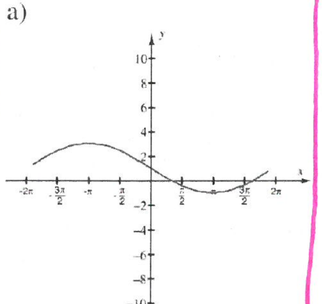
e. Find the amplitude 2

f. Find the period 4π

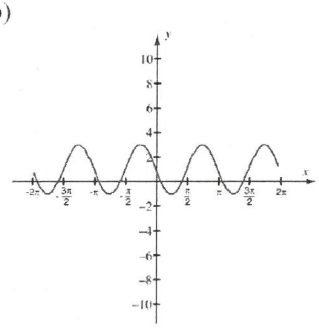
g. Find the phase shift -2π

h. Which is the graph of the function? a

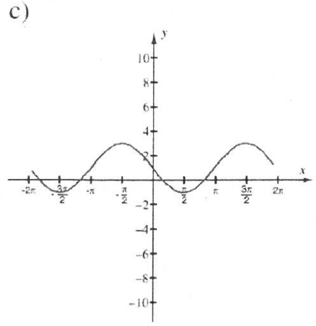
a)



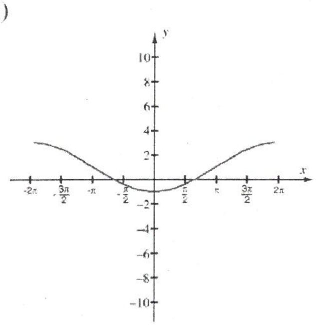
b)



c)



d)



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