Assignments for the Week of Sept. 26

- Read 6.1-6.3, 6.5
- 45 minutes of Khan Academy
- Textbook assignment due Friday, Sept. 21:
  6.1 #1-69 odd (proofs)

- Textbook assignment due Test Day:
  6.3 #1-24 all; 30-36 all; 49-93 odd
- **TEST #3**
  9:00 class - Tues. 10/4
  8:00 class - Wed. 10/5

Coming up after the test:

- 6.5 #1-24 all #25-55 odd Inverse Trig Functions
- 6.6 Solving Trig Equations

$$a^3+b^3=(a+b)(a^2-ab+b^2)$$

$$\frac{\sin^3 x+\cos^3 x}{\sin x+\cos x}=\frac{(\cancel{\sin x+\cos x})(\sin^2 x-\sin x\cos x+\cos^2 x)}{\cancel{\sin x+\cos x}}$$

$$=\sin^2 x+\cos^2 x-\sin x\cos x=1-\sin x\cos x$$

Review

$\cos(105^\circ) = \cos(60^\circ + 45^\circ)$

$= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$

$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{2} - \sqrt{6}}{4}}$

$\tan\left(\frac{\pi}{8}\right) = \tan \frac{\pi/4}{2} = \frac{1 - \cos \pi/4}{\sin \pi/4}$

$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}$

$= \frac{1 - \frac{\sqrt{2}}{2}}{\frac{1}{\sqrt{2}}} = \left(1 - \frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{1}\right)$

$= \boxed{\sqrt{2} - 1}$

86. $\frac{\cos 2x}{\sin^2 x} = \csc^2 x - 2$

$LHS = \frac{1 - 2\sin^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} - \frac{2\sin^2 x}{\sin^2 x} = RHS$

88. $$\frac{2\cos 2x}{\sin 2x} = \cot x - \tan x$$

$$RHS = \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}$$

$$= \frac{\cos x}{\sin x} \cdot \frac{\cos x}{\cos x} - \frac{\sin x}{\cos x} \cdot \frac{\sin x}{\sin x}$$

$$= \frac{\cos^2 x - \sin^2 x}{\sin x \cos x}$$

$$= \frac{\cos 2x}{\sin x \cos x} = \frac{2\cos 2x}{2\sin x \cos x}$$

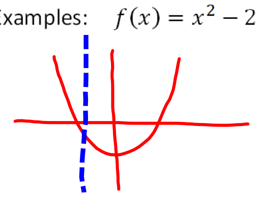
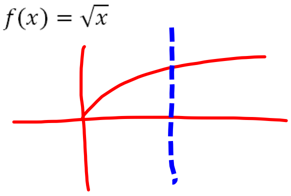
$$= \frac{2\cos 2x}{\sin 2x} = LHS$$

$\sin 2\theta = 2\sin\theta\cos\theta$

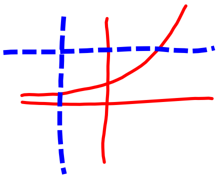
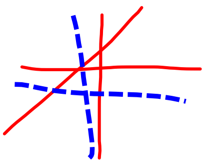
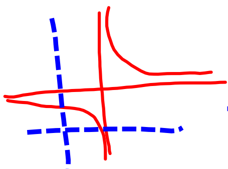
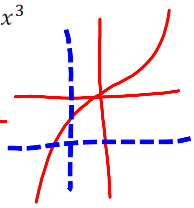
Inverse Trigonometric Functions

Recall from Algebra:

- f is a **function** if each input value (x) has a unique output $f(x)$.
  Examples: $f(x) = x^2 - 2$, $f(x) = \sqrt{x}$

- f is **one-to-one** if, in addition, each y corresponds to only one x.
  Examples: $y = e^x$, $y = x + 3$, $y = \frac{1}{x}$, $y = x^3$

- If f is a one-to-one function, we can define its inverse $f^{-1}(x)$.
  Note that this notation is not exponentiation, i.e. $f^{-1}(x) \neq \frac{1}{f(x)}$
- $f(x)$ and $g(x)$ are **inverses** if
  $(f \circ g)(x) = f(g(x)) = x = g(f(x)) = (g \circ f)(x)$,
  that is, **inverse functions "undo" each other**.

$x^{-n} = \frac{1}{x^n}$

Example: $f(x) = x^3$, $g(x) = \sqrt[3]{x}$

$(f \circ g)(x) = (\sqrt[3]{x})^3 = x$

$(g \circ f)(x) = \sqrt[3]{x^3} = x$

What do we mean by an Inverse Trig function?

Recall that **for a basic Trigonometric function**, e.g. $f(x) = \sin x$,

- **The input (x) is an angle**
- **The output $f(x)$ is a ratio of sides**

So **for an inverse Trigonometric function,**

- **The input (x) is a ratio of sides**
- **The output $f(x)$ is an angle**

Construction of the inverse of $f(x) = \sin x$:

$f(x) = x^3 - 8$

$y = x^3 - 8$

$x = y^3 - 8$

$x + 8 = y^3$

$\sqrt[3]{x+8} = y$

$f^{-1}(x) = \sqrt[3]{x+8}$

$y = \sin x$

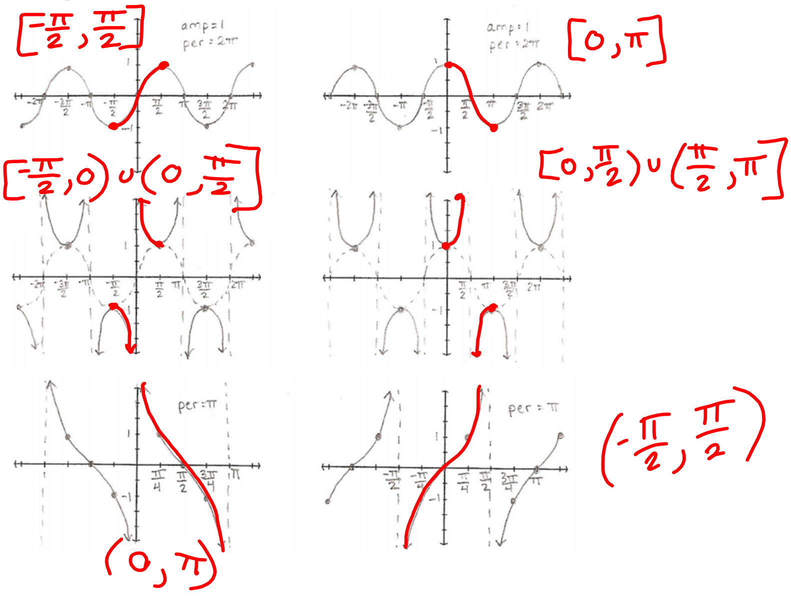
$x = \sin y$

$y =$ the angle whose sine value is x

$y = \sin^{-1} x$

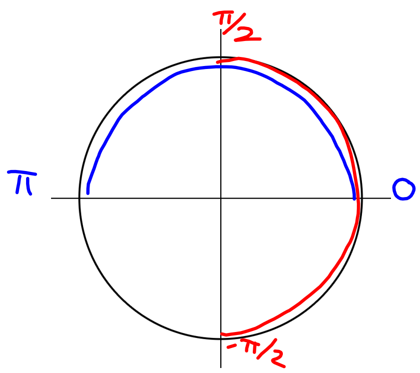
(or $y = \arcsin x$)

But Trigonometric functions aren't one-to-one – how is the inverse defined? We must restrict the domain!



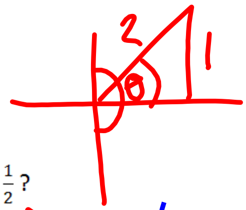
Summary of Restricted Domains:

Interval	Functions	Quadrants
$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$	$\sin x$, $\csc x$, $\tan x$	IV & I
$(0,\pi)$	$\cos x$, $\sec x$, $\cot x$	I & II



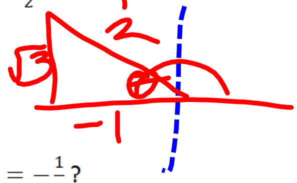
Evaluate the inverse trigonometric expression.

$\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$



In words: What angle θ, between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ (the restricted domain for sine) is such that $\sin\theta = \frac{1}{2}$?

$\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$



In words: What angle θ, between 0 and π (the restricted domain for cosine) is such that $\cos\theta = -\frac{1}{2}$?

$\tan^{-1}(-1) = -\frac{\pi}{4}$

$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$