

Assignments for the Week of Oct. 17

- Read 7.1-7.3
- 45 minutes of Khan Academy
- Textbook assignment due Friday 10/21:
 

7.1 #1-21 odd; 29,30,33,34,35	Law of Sines
• 7.2 #9-19 odd #25-29 odd; #38,43,46,47,48	Law of Cosines Area Law of Cosines
• 7.3 #37,41,43	word problems with Law of Sines/Cosines

**Test #4 - Friday, 10/21**

$$1. \cos^{-1}\left(-\frac{1}{2}\right) = \boxed{\frac{2\pi}{3}}$$

$$2. \sin \frac{7\pi}{4} = \boxed{-\frac{1}{\sqrt{2}}} = -\frac{\sqrt{2}}{2}$$

$$3. \cot^{-1}\left(\cot \frac{2\pi}{3}\right) = \boxed{\frac{2\pi}{3}}$$

$$4. \csc \frac{4\pi}{3} = \boxed{-\frac{2}{\sqrt{3}}} = -\frac{2\sqrt{3}}{3}$$

$$5. \sec^{-1}(-\sqrt{2}) = \boxed{\frac{3\pi}{4}}$$

$$\cot^{-1}\left(\cot \frac{2\pi}{3}\right) = \frac{2\pi}{3}$$

$$\cot^{-1}\left(\cot \frac{4\pi}{3}\right) = \frac{\pi}{3}$$

$$2\sin^2 x = \sin x$$

$$2\sin^2 x - \sin x = 0$$

$$\sin x (2\sin x - 1) = 0$$

$$\sin x = 0, \quad 2\sin x - 1 = 0$$

$$x = 0, \pi$$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$1 - 4\cos^2 x = 0$$

$$(1 - 2\cos x)(1 + 2\cos x) = 0$$

$$\cos x = \frac{1}{2}, \cos x = -\frac{1}{2}$$

$$1 = 4\cos^2 x$$

$$\frac{1}{4} = \cos^2 x$$

$$\pm \frac{1}{2} = \cos x$$

$$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

### The Law of Sines

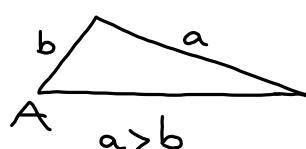
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

or

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

### ASS , The Problematic Triangle

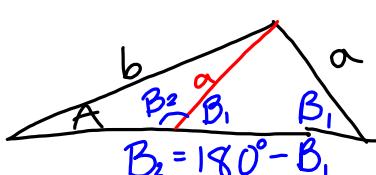
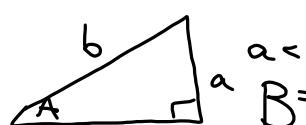
one solution:



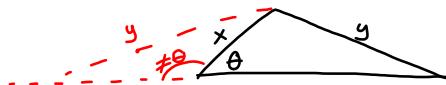
no solutions:



two solutions:  $a < b$



Why does this ASS triangle have only one solution?



The measure of  $\theta$  and the lengths of  $x$  &  $y$  are fixed. If we try to reposition  $y$ , the measure of  $\theta$  changes, unlike in the 2-solution case:

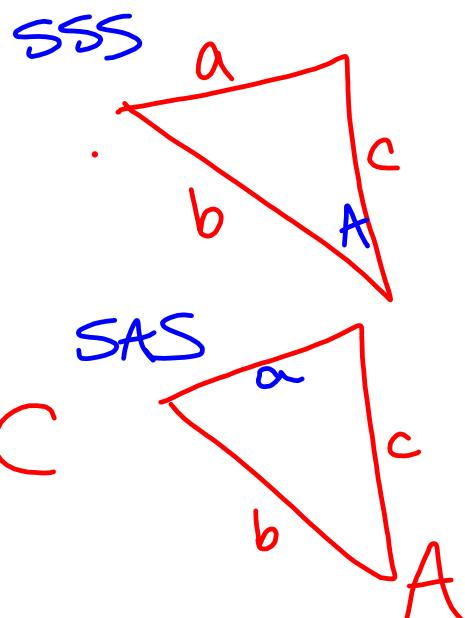


### The Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$



7.2  $\text{SSS} \Rightarrow \text{Law of Cosines}$ 

16.  $a = 60, b = 88, c = 120. B = ?$

$$b^2 = a^2 + c^2 - 2ac \cdot \cos B$$

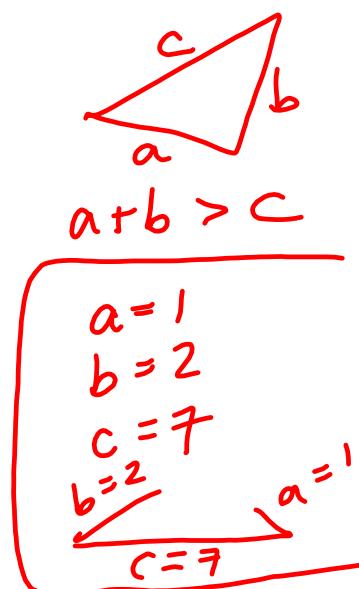
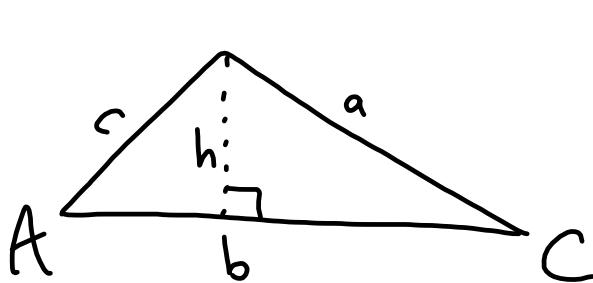
$\underbrace{b^2}_{\rightarrow} = \underbrace{a^2 + c^2}_{\leftarrow} - \underbrace{2ac \cdot \cos B}_{\rightarrow}$

$$2ac \cos B = a^2 + c^2 - b^2$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

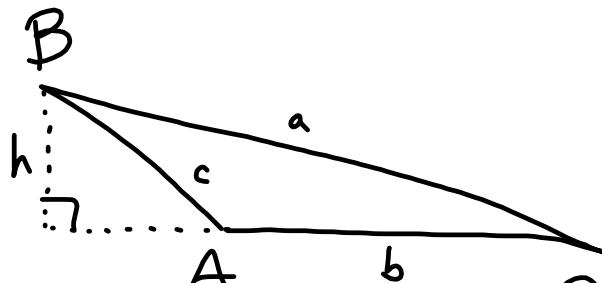
$$B = \cos^{-1} \left( \frac{a^2 + c^2 - b^2}{2ac} \right)$$

$$= \cos^{-1} \left( \frac{60^2 + 120^2 - 88^2}{2(60)(120)} \right) \approx 44.6^\circ$$

7.1/7.2 Area of a Triangle

Acute

$$\begin{aligned}\sin A &= \frac{h}{c} \\ c \sin A &= h \\ \text{area} &= \frac{1}{2} b \cdot c \sin A\end{aligned}$$



Obtuse

$$\begin{aligned}\sin A &= \frac{h}{c} \\ c \sin A &= h \\ \text{area} &= \frac{1}{2} b \cdot c \sin A\end{aligned}$$

# Area of a $\triangle$

$$K = \frac{1}{2}bc \sin A$$

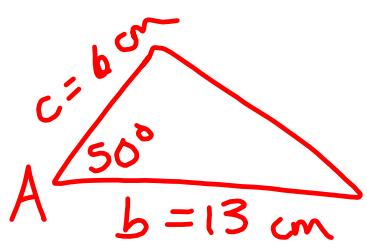
$$K = \frac{1}{2}ac \sin B$$

$$K = \frac{1}{2}ab \sin C$$

$\frac{1}{2}$  the product  
of 2 sides &  
the sine of the  
included angle

Find the area of the triangle.

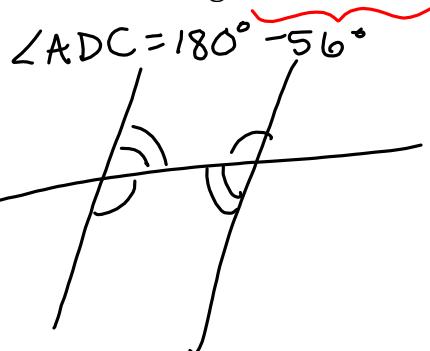
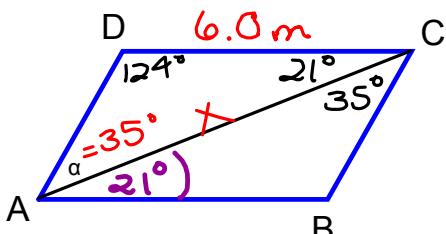
$$A = 50^\circ, b = 13 \text{ cm}, c = 6 \text{ cm}$$



$$\begin{aligned} \text{area} &= \frac{1}{2}(6 \text{ cm})(13 \text{ cm}) \sin 50^\circ \\ &= [29.9 \text{ cm}^2] \end{aligned}$$

7.1 #28

The longer side of a parallelogram is 6.0 meters. The measure of angle BAD is  $56^\circ$  and  $\alpha$  is  $35^\circ$ . Find the length of the longer diagonal.

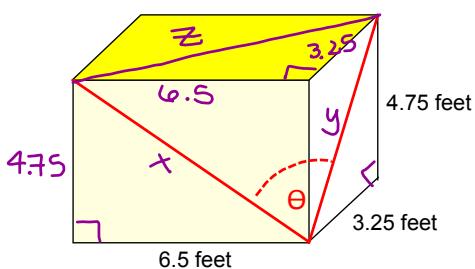


$$\frac{x}{\sin 124^\circ} = \frac{6}{\sin 35^\circ}$$

$$x = \frac{6 \sin 124^\circ}{\sin 35^\circ} = 8.7 \text{ m}$$

7.2 #41

The rectangular box in the figure measures 6.50 feet by 3.25 feet by 4.75 feet. Find the measure of the angle  $\theta$  that is formed by the union of the diagonal shown on the front of the box and the diagonal shown on the right side of the box.



$$x^2 = 4.75^2 + 6.5^2$$

$$x = \sqrt{4.75^2 + 6.5^2} = 8.05$$

$$y^2 = 3.25^2 + 4.75^2$$

$$y = \sqrt{3.25^2 + 4.75^2} = 5.76$$

$$z^2 = 6.5^2 + 3.25^2$$

$$z = \sqrt{6.5^2 + 3.25^2} = 7.27$$

$$z^2 = x^2 + y^2 - 2xy \cos \theta$$

$$2xy \cos \theta = x^2 + y^2 - z^2$$

$$\cos \theta = \frac{x^2 + y^2 - z^2}{2xy}$$

$$\theta = \cos^{-1} \left( \frac{x^2 + y^2 - z^2}{2xy} \right) = \cos^{-1} \left( \frac{8.05^2 + 5.76^2 - 7.27^2}{2(8.05)(5.76)} \right)$$

$$\theta = 60.88^\circ$$