Assignments for the week of Oct 24:

- 45 minutes of Khan Academy
- Textbook problems due Wed(2nd)/Thurs(1st)

7.3	#1-23 odd, 33,35	vector operations
	#45-51 odd	dot product

Final Exams

1st period: Wed. 11/2 9am-11am

2nd period: Fri. 10/28 9am-11am

Bring your textbooks to class on the last day: Wed(2nd)/Thurs(1st)
It will be up to you to turn them in to the library.

EXTRA CREDIT OPPORTUNITY:
Math Competition TONIGHT (Mon, 10/24) @ 5:00pm

10. $\tan^2 2x = \frac{1}{3}$

$\tan 2x = \pm\frac{1}{\sqrt{3}}$

$2x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}$

$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

$$2\sin^2 x - \sin x = 0$$

$$\sin x(2\sin x - 1) = 0$$

$$\sin x = 0 \qquad \sin x = \frac{1}{2}$$

$$\boxed{x = 0, \pi \qquad x = \frac{\pi}{6}, \frac{5\pi}{6}}$$

$$\cos x \sin x + \cos x - \sin x - 1 = 0$$

$$\cos x(\sin x + 1) - 1(\sin x + 1) = 0$$

$$(\sin x + 1)(\cos x - 1) = 0$$

$$\sin x = -1, \quad \cos x = 1$$

$$\boxed{x = \frac{3\pi}{2}, 0}$$

13. SSS $A = \cos^{-1}\left(\frac{20^2+31^2-19^2}{2(20)(31)}\right) = \boxed{36.2^\circ}$

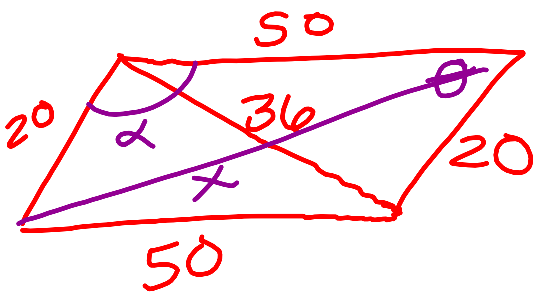
14. $a = \frac{37\sin 42^\circ}{\sin 124^\circ} = \boxed{29.9}$

15. $a = \sqrt{51^2+34^2-2(51)(34)\cos 40^\circ} = \boxed{33.2}$

16. ASS

$A = \sin^{-1}\left(\frac{9\sin 18^\circ}{4}\right) = \boxed{44.1^\circ}$

case 2: $A = 180^\circ - 44.1^\circ = \boxed{135.9^\circ}$



$$36^2 = 20^2 + 50^2 - 2(20)(50)\cos\theta$$

$$\theta = 36.7^\circ$$

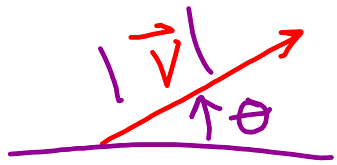
$$\alpha = 180^\circ - \theta$$

$$x = \sqrt{20^2 + 50^2 - 2(20)(50)\cos 143.3^\circ}$$

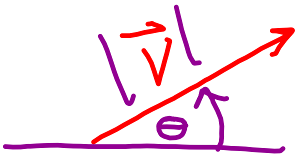
$$= \boxed{67.1}$$

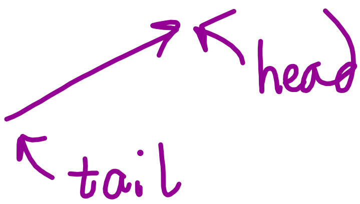
$\vec{v}$ $\hat{v}$ $\mathbf{v}$

Vectors

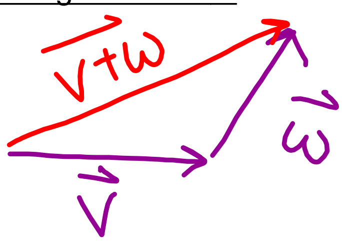


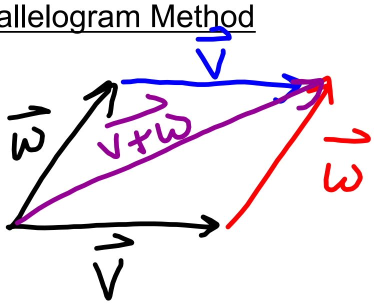
A vector is a directed line segment;
it has a unique length (magnitude) and direction angle





Vector Addition:
Vectors can be added using the
Triangle Method or the Parallelogram Method



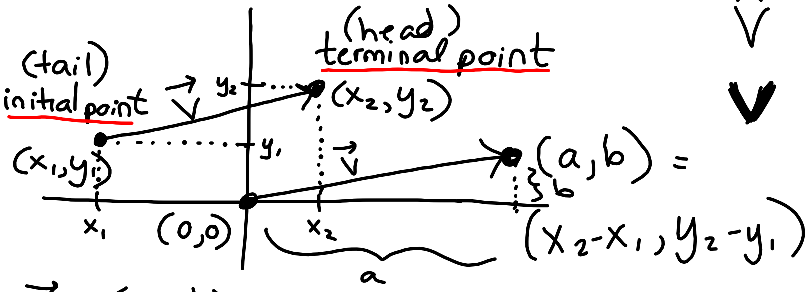


Vectors!

$\vec{v}$

$\hat{v}$

$\mathbf{v}$



$\vec{v} = \langle a, b \rangle$ = "component form" of the vector whose initial point is (0,0) and terminal point is (a,b).

$\overrightarrow{CD}$, C(2,5) , D(3,-1)

initial pt   terminal pt

find a vector $\vec{v}$ equivalent to $\overrightarrow{CD}$ whose initial point is (0,0).

terminal point − initial point

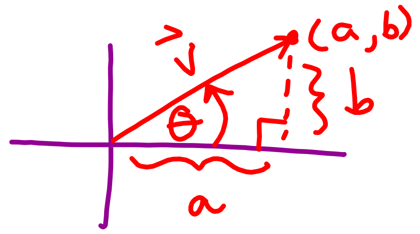
$(3-2, -1-5) = (1, -6)$

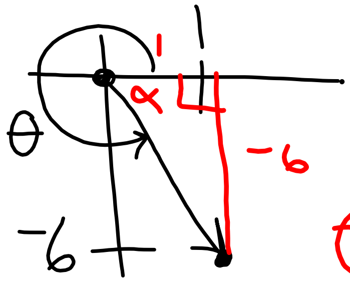
$$\boxed{\vec{v} = \langle 1, -6 \rangle}$$

magnitude of $\vec{v} = \langle a, b \rangle$

$$|\vec{v}| = \sqrt{a^2+b^2}$$

direction $\angle\theta$ is measured counter-clockwise from $0°$.





$\vec{w} = \langle 1, -6 \rangle$

$\tan\alpha = \left|\frac{-6}{1}\right|$ $\quad \alpha = \tan^{-1}(|-6|)$

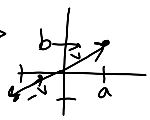
$= \tan^{-1}(6) \approx 80.5°$

$\theta = 360° - 80.5° = \boxed{279.5°}$

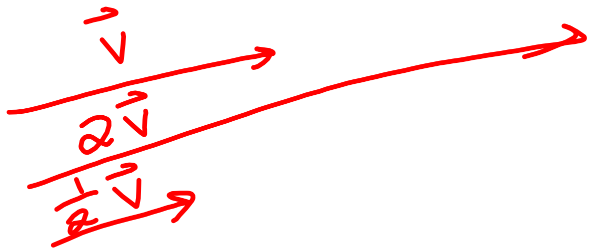
Vector Operations

$\vec{v} = \langle a, b \rangle$; $\vec{w} = \langle c, d \rangle$; $k \in \mathbb{R}$

1. $|\vec{v}| = \sqrt{a^2+b^2}$
2. $k\vec{v} = k\langle a, b \rangle = \langle ka, kb \rangle$
   "scalar multiplication"
3. $\vec{v} + \vec{w} = \langle a+c, b+d \rangle$
4. $-\vec{v} = \langle -a, -b \rangle$



5. $\vec{v} - \vec{w} = \langle a-c, b-d \rangle$
6. $\vec{0} = \langle 0, 0 \rangle$ "zero vector"



$\vec{v} = \langle 12, -5 \rangle$; $\vec{w} = \langle 2, 7 \rangle$

a. $|\vec{v}| = \sqrt{(12)^2 + (-5)^2} = 13$

b. $\vec{v} + \vec{w} = \langle 12+2, -5+7 \rangle = \langle 14, 2 \rangle$

c. $-5\vec{v} = -5\langle 12, -5 \rangle = \langle -60, 25 \rangle$

d. $3\vec{v} - 4\vec{w} = 3\langle 12, -5 \rangle - 4\langle 2, 7 \rangle$
$= \langle 36, -15 \rangle - \langle 8, 28 \rangle = \langle 28, -43 \rangle$

vector multiplication

$\vec{v} \cdot \vec{w}$ vs. $\vec{v} \times \vec{w}$

dot product — cross product

result is a scalar (#) — result is a vector

$\vec{v} = \langle a, b \rangle$; $\vec{w} = \langle c, d \rangle$

$\vec{v} \cdot \vec{w} = ac + bd$

$\vec{v} = \langle 1,2 \rangle$, $\vec{w} = \langle -3, 4 \rangle$

$\vec{v} \cdot \vec{w} = 1(-3) + 2(4) = -3 + 8 = \boxed{5}$

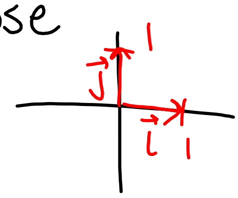
$\vec{v}_1 = \langle 1,2 \rangle$, $\vec{v}_2 = \langle -3,4 \rangle$, $\vec{v}_3 = \langle 5,-6 \rangle$

$\vec{v}_1 \cdot (\vec{v}_2 + \vec{v}_3) = \langle 1,2 \rangle \cdot \langle 2,-2 \rangle$

$= 1(2) + 2(-2) = 2 - 4 = \boxed{-2}$

7.6 Unit Vectors

A <u>unit vector</u> is a vector whose magnitude is 1.



special unit vectors:

$\vec{i} = \langle 1,0 \rangle$ & $\vec{j} = \langle 0,1 \rangle$

If 3 dimensions...

$\vec{i} = \langle 1,0,0 \rangle$; $\vec{j} = \langle 0,1,0 \rangle$; $\vec{k} = \langle 0,0,1 \rangle$

$\vec{v} = \langle a, b \rangle$ "component form"

$= \langle a, 0 \rangle + \langle 0, b \rangle$

$= a\langle 1, 0 \rangle + b\langle 0, 1 \rangle$

$\vec{v} = a\vec{i} + b\vec{j}$

$\vec{u} = 2\vec{i} + \vec{j}$; $\vec{v} = -3\vec{i} - 10\vec{j}$; $\vec{w} = \vec{i} - 5\vec{j}$

46. $\vec{v} + 3\vec{w} = -3\vec{i} - 10\vec{j} + 3(\vec{i} - 5\vec{j})$

$= -3\vec{i} - 10\vec{j} + 3\vec{i} - 15\vec{j}$

$= -25\vec{j} = \langle 0, -25 \rangle$

$\vec{v} = 3\vec{i} - 2\vec{j}$

$|\vec{v}| = \sqrt{a^2 + b^2}$

$= \sqrt{3^2 + (-2)^2}$

$= \sqrt{9+4}$

$= \boxed{\sqrt{13}}$

48. $(\vec{u} - \vec{v}) + \vec{w}$

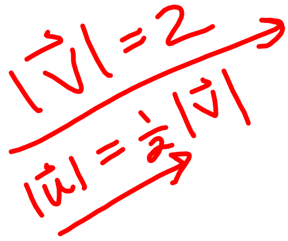
$(2\vec{i} + \vec{j} - (-3\vec{i} - 10\vec{j})) + \vec{i} - 5\vec{j}$

$= 5\vec{i} + 11\vec{j} + \vec{i} - 5\vec{j}$

$= \boxed{6\vec{i} + 6\vec{j}} = \langle 6, 6 \rangle$

Given a vector $\vec{v} = \langle a, b \rangle$ we can find a unit vector $\vec{u}$ in the direction of $\vec{v}$ by dividing each component by $|\vec{v}|$.

$$\vec{u} = \left\langle \frac{a}{\sqrt{a^2+b^2}}, \frac{b}{\sqrt{a^2+b^2}} \right\rangle$$



$\vec{v} = \langle -3, 4 \rangle$

Find a unit vector $\vec{u}$ in the direction of $\vec{v}$.

$$|\vec{v}| = \sqrt{(-3)^2 + 4^2} = 5$$

$$\vec{u} = \boxed{\left\langle \frac{-3}{5}, \frac{4}{5} \right\rangle}$$

verify: $|\vec{u}| = \sqrt{\left(\frac{-3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} = \sqrt{\frac{9}{25} + \frac{16}{25}}$

$$= \sqrt{\frac{25}{25}} = \sqrt{1} = 1$$