$f(x) = -3x^2 - 12x + 2 = -3(x^2+4x+4)+12+2$

$= -3(x+2)^2 + 14$

vertex: $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$ $(-2, 14)$

axis of symmetry: $x=-2$

opens upward / downward; vertex is maximum / minimum

domain: $(-\infty, \infty)$

range: $(-\infty, 14]$

y-intercept: $(0, 2)$

x-intercepts: $-3x^2-12x+2=0$

$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(-3)(2)}}{2(-3)} = \frac{12 \pm \sqrt{144+24}}{-6} = \frac{12 \pm \sqrt{168}}{-6}$

$= \frac{12 \pm 2\sqrt{42}}{-6} = -2 \pm \frac{\sqrt{42}}{3}$

$\left(-2+\frac{\sqrt{42}}{3}, 0\right), \left(-2-\frac{\sqrt{42}}{3}, 0\right)$

Turn in HW #5

- **2.4: #3-13 odd** Find vertex, etc. and graph
  #29-37 odd Find vertex, etc.
  #41-49 odd Applications of quadratics
- **3.1: #8-14 all**
  Describing simple characteristics of polynomials
  #23-38 all
  Determining zeros & multiplicities of polynomials
- **3.2: #16,17,21,22-32all** Graph polynomials
- **3.3: #7,9,13,19,21,23, 35**
  Long & synthetic polynomial division

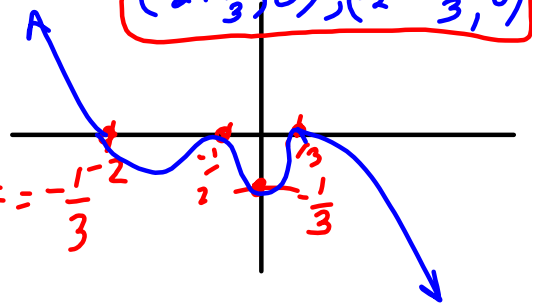
Graph: $f(x) = -6\left(x+\frac{1}{2}\right)^2\left(x-\frac{1}{3}\right)^2(x+2)$

zeros: $-1/2$, $1/3$, -2

multiplicity: (2) (2) (1)

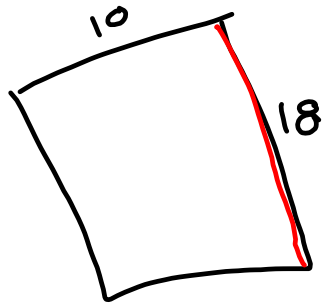
y-intercept: $-6\left(\frac{1}{2}\right)^2\left(-\frac{1}{3}\right)^2(2) = -\frac{6}{1}\cdot\frac{1}{4}\cdot\frac{1}{9}\cdot\frac{2}{1} = -\frac{1}{3}$

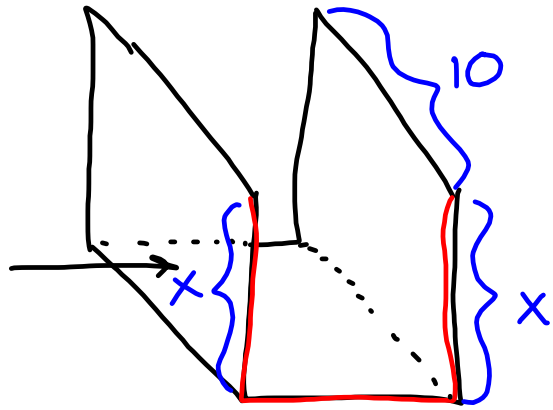
lead term: $-6x^5$



$x + w + x = 18$

$w = 18 - 2x$





$l = 10$

$h = x$

$w = ?$

What height maximizes the volume?

$V = lwh = 10x(18-2x)$

$V(x) = -20x^2 + 180x$

$\frac{-b}{2a} = \frac{-180}{2(-20)} = \frac{18\cancel{0}}{4\cancel{0}} = \frac{9}{2} = 4.5''$ — height that yields max volume

max volume

$-20\left(\frac{9}{2}\right)^2 + 180\frac{9}{2}$

$-\cancel{20}^{5} \cdot \frac{81}{4} + 90(9)$

$-405 + 810 = 405 \text{ in}^3$

Descartes' Rule of Signs

If $P(x)$ is written in descending order with real numbered coefficients and a nonzero constant term*, then

- The number of positive real zeros is either
  - the number of sign changes of $P(x)$ or
  - less than that number by a positive even integer.
- The number of negative real zeros is either
  - the number of sign changes of $P(-x)$ or
  - less than that number by a positive even integer.

* if the constant term is zero, factor out x (or GCF), and use Descartes' rule of signs on whatever's leftover

3.4

Descartes' Rule of Signs

84. $H(t) = 5t^{12} - 7t^4 + 3t^2 + t + 1$

$+ \quad - \quad + \quad + \quad +$

2 sign changes
$\Rightarrow$ either 2 or 0 positive real zeros

$H(-t) = 5(-t)^{12} - 7(-t)^4 + 3(-t)^2 + (-t) + 1$

$H(-t) = 5t^{12} - 7t^4 + 3t^2 - t + 1$

4 sign changes $\Rightarrow$ either 4 or 2 or 0 negative real zeros

Descartes' Rule of Signs

80. $g(x) = 5x^6 - 3x^3 + x^2 - x$

$= x(5x^5 - 3x^2 + x - 1)$

0 is one of our zeros

3 sign changes $\Rightarrow$ 3 or 1 positive real zeros

$g(-x) = -x[5(-x)^5 - 3(-x)^2 + (-x) - 1]$

$= -x[-5x^5 - 3x^2 - x - 1]$

0 sign changes $\Rightarrow$ no negative real zeros

Descartes' Rule of Signs

86. $g(z) = -z^{10} + 8z^7 + z^3 + 6z - 1$

2 sign changes $\Rightarrow$ 2 or 0 positive real zeros

$g(-z) = -(-z)^{10} + 8(-z)^7 + (-z)^3 + 6(-z) - 1$

$= -z^{10} - 8z^7 - z^3 - 6z - 1$

0 sign changes $\Rightarrow$ no negative real zeros

Factor:

$$f(x) = x^3 + 3x^2 - 2x - 6$$

$$= x^2(x+3) - 2(x+3)$$

$$= (x+3)(x^2-2)$$

$$= \boxed{(x+3)(x-\sqrt{2})(x+\sqrt{2})}$$

$x^2 - 2 = 0$

$x^2 = 2$

$x = \pm\sqrt{2}$

zeros: $-3, \sqrt{2}, -\sqrt{2}$

3.4

Find all the zeros and write f(x) in factored form.

63. $f(x) = x^4 - 3x^3 - 20x^2 - 24x - 8$

-1	1	-3	-20	-24	-8
		-1	4	16	8
	1	-4	-16	-8	0

$f(x) = (x+1)(x^3 - 4x^2 - 16x - 8)$

-2	1	-4	-16	-8
		-2	12	8
	1	-6	-4	0

$f(x) = (x+1)(x+2)(x^2 - 6x - 4)$

$x^2 - 6x - 4 = 0$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-4)}}{2(1)} = \frac{6 \pm \sqrt{36+16}}{2} = \frac{6 \pm \overset{4\sqrt{13}}{\sqrt{52}}}{2} = 3 \pm 2\sqrt{13}$$

$$\boxed{f(x) = (x+2)(x+1)(x - (3+2\sqrt{13}))(x - (3-2\sqrt{13}))}$$

<u>HW #6</u> (due Fri, 09/12)

- <u>3.4</u>: #7-16all Given the zeros of a polynomial, find the polynomial
  #25-32all; 43-47odd Given some zeros of a polynomial, find the other zeros
  #51-54all List all possible rational zeros
  #55-69odd Find all the zeros and write f(x) in factored form
  #79,89,93 Descartes' rule of signs
  #95-98all Graph the polynomial
- <u>3.5</u>: #7-25odd Determining asymptotes of rational functions
  #27-67odd Graphing rational functions

Quiz #3 - Wed, 09/10
Test #2 - Mon, 09/15