Precalculus Review

Rational Functions

$$f(x) = \frac{p(x)}{q(x)}, \quad p(x), q(x) - \text{polynomials}$$

<u>zeros of the function</u> – input (x) value
such that $f(x) = 0$
  ✳ specifically, x's that make
  numerator $p(x) = 0$

<u>x-intercepts</u> – the points $(x, 0)$
  (where x ~~is x = 0 function~~ on the
  graph of the function (where the
  graph crosses the x-axis)

<u>y-intercept</u> – the point $(0, f(0))$
  found by plugging in 0

<u>vertical asymptotes</u>: vertical line through
an x-value where the function is undefined;
the graph of the function approaches the
line but never touches it

as the x-values approach
the undefined x-value,
$f(x)$ approaches $\pm\infty$.

$$\text{as } x \to c, \quad f(x) \to \pm\infty$$
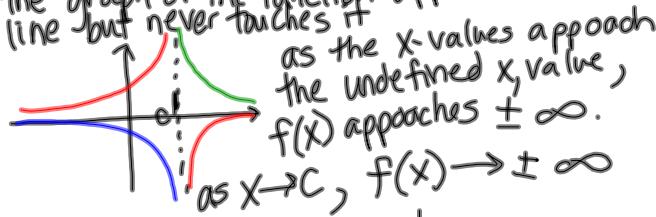
$(\to \pm\infty \equiv$ increases or decreases
  without bound$)$

<u>horizontal asymptotes</u> – determine the
  end behavior of a function
  (what happens to
  $f(x)$ as $x \to \pm\infty$)
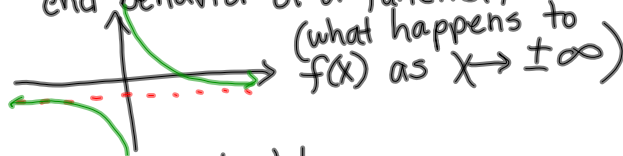
look at the lead terms
<u>Case 1</u> – degrees are same
$$f(x) = \frac{3x^5 - 5x^2 + 2}{6x^5 + 3x^3 - x} \qquad \frac{3x^5}{6x^5} \approx \frac{1}{2}$$
  ratio of lead terms:

$$\text{as } x \to \pm\infty, \quad f(x) \to \frac{1}{2}$$