

## Horizontal Asymptotes

Case 1 degrees same



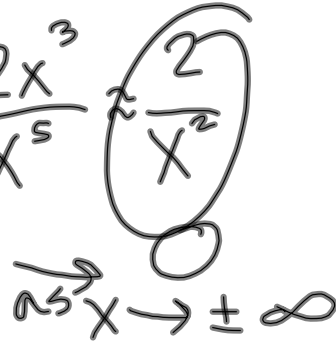
H.A. :  $y = \text{ratio of leading coefficients}$

e.g.  $\frac{3x^5 - 2x^4 + 3}{922x^5 - 3x^2} \approx \frac{3x^5}{922x^5} \approx \frac{3}{922}$

Case 2 degree of denominator is larger than deg. of numerator

e.g.  $\frac{2x^3 - 3x^2 + x}{x^5 - 2x + 7} \approx \frac{2x^3}{x^5} \approx \frac{2}{x^2}$

H.A. is always  $y = 0$



Case 3 Oblique Asymptote

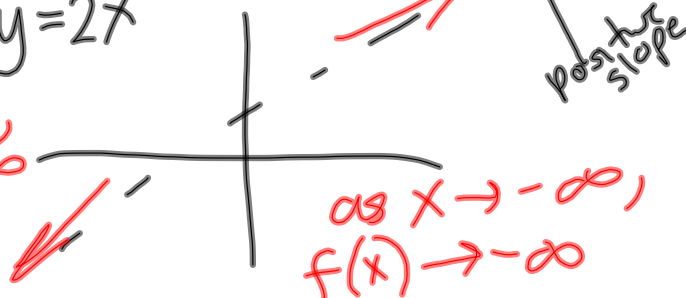
deg. of numerator is larger than deg of denominator

e.g.  $\frac{2x^5 - 3x^3 + 2x^2 - 7}{x^4 - 1} \approx \frac{2x^5}{x^4} \approx 2x$

O.A.  $\approx y = 2x$

as  $x \rightarrow +\infty$ ,  
 $f(x) \rightarrow +\infty$

as  $x \rightarrow -\infty$ ,  
 $f(x) \rightarrow -\infty$



$$\frac{-2x^7 + 3x^4 - 2}{4x^3 - x} \approx \frac{-2x^7}{4x^3} = \frac{-x^4}{2}$$

negative  
 even degree

$y = f(x)$

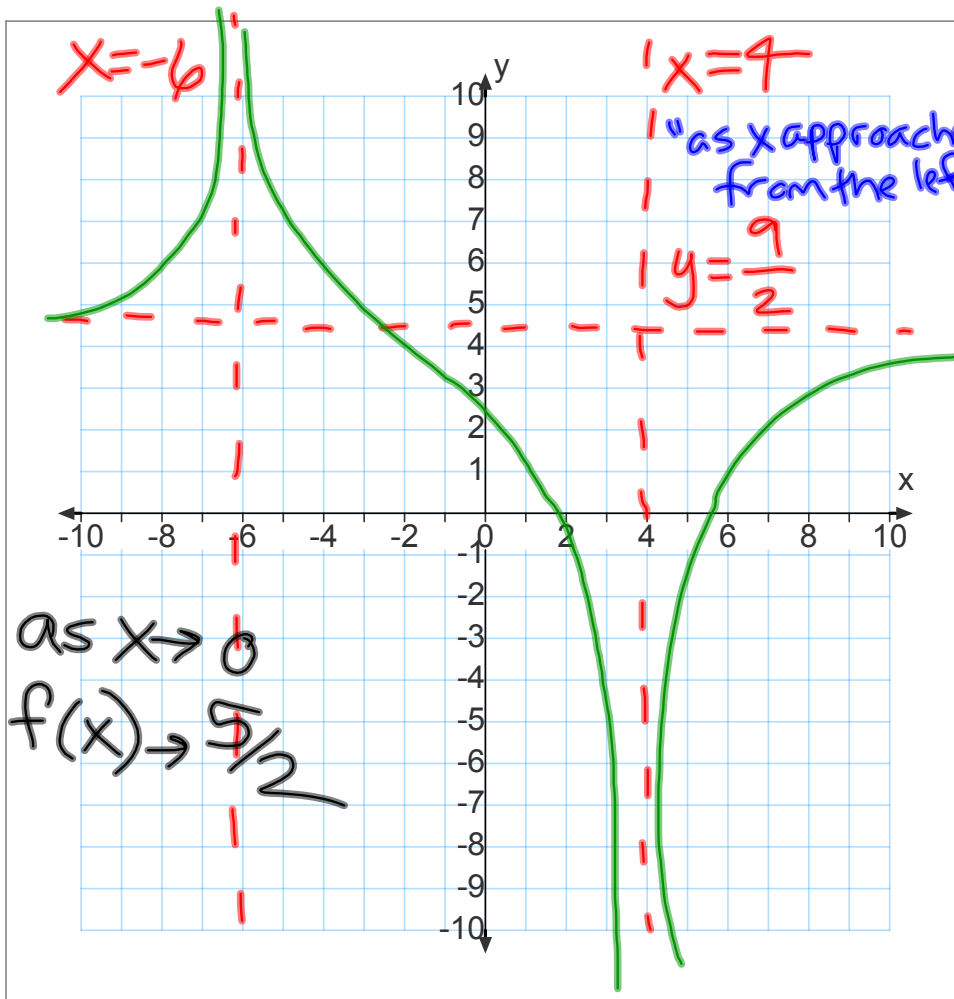
$-\infty$

$x$

$\infty$

as  $x \rightarrow \infty$ ,  $f(x) \rightarrow -\infty$

as  $x \rightarrow -\infty$ ,  $f(x) \rightarrow -\infty$



$x = -6$

$x = 4$

"as x approaches -6 from the left"

$y = \frac{9}{2}$

as  $x \rightarrow -6^-$   
 $f(x) \rightarrow \infty$

as  $x \rightarrow -6^+$   
 $f(x) \rightarrow \infty$

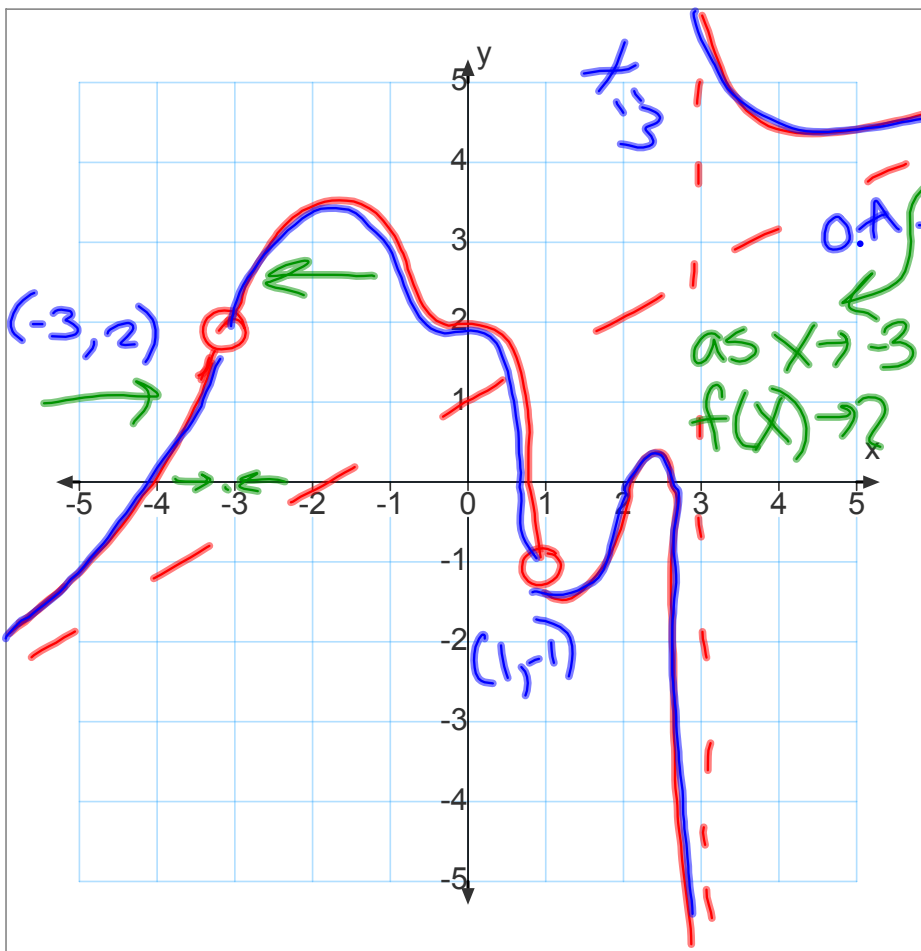
as  $x \rightarrow 4^-$   
 $f(x) \rightarrow -\infty$

as  $x \rightarrow 4^+$   
 $f(x) \rightarrow -\infty$

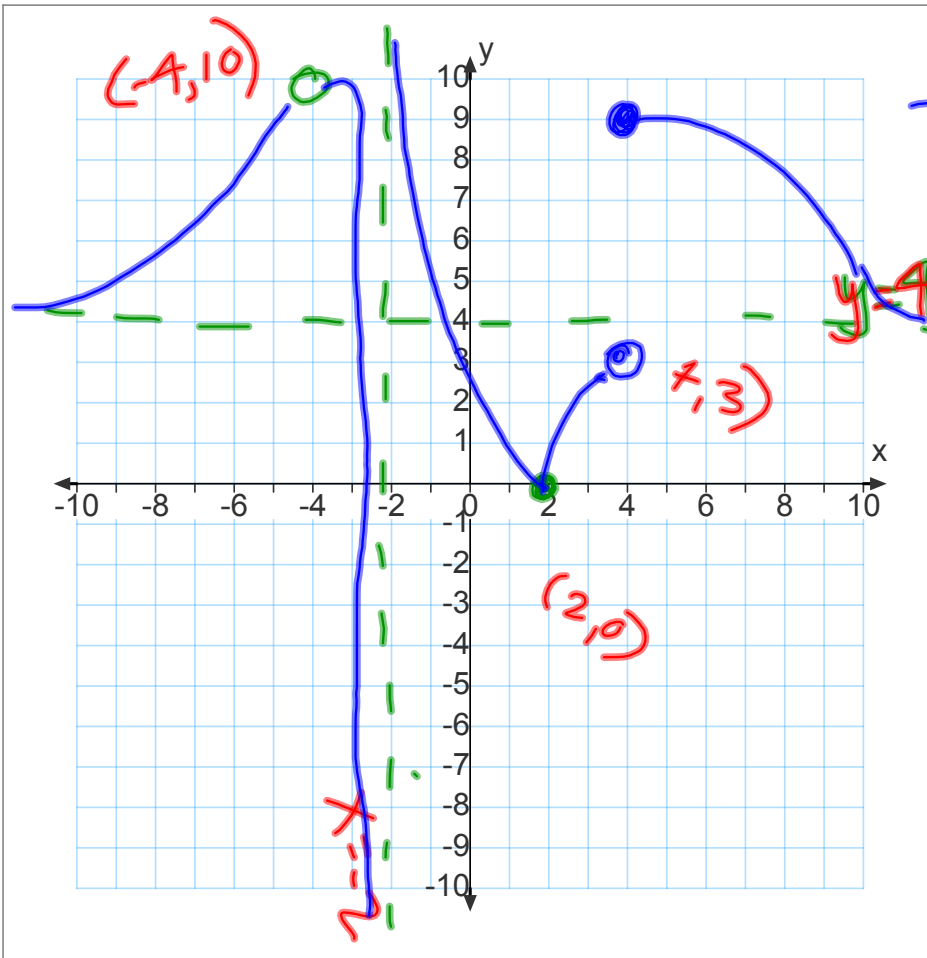
as  $x \rightarrow \infty$   
 $f(x) \rightarrow \frac{9}{2}$

as  $x \rightarrow -\infty$   
 $f(x) \rightarrow \frac{9}{2}$

as  $x \rightarrow 0$   
 $f(x) \rightarrow \frac{9}{2}$



$\left. \begin{array}{l} \text{as } x \rightarrow -3^- \\ f(x) \rightarrow 2 \\ x \rightarrow -3^+ \\ f(x) \rightarrow 2 \end{array} \right\}$   
 $\left. \begin{array}{l} x \rightarrow 1^- \\ f(x) \rightarrow -1 \\ x \rightarrow 1^+ \\ f(x) \rightarrow -1 \end{array} \right\}$   
 $\left. \begin{array}{l} x \rightarrow 3^- \\ f(x) \rightarrow -\infty \\ x \rightarrow 3^+ \\ f(x) \rightarrow +\infty \end{array} \right\}$   
 $\left. \begin{array}{l} x \rightarrow -\infty \\ f(x) \rightarrow -\infty \end{array} \right\}$



$x \rightarrow$	$f(x) \rightarrow$
$-4^-$	10
$-4^+$	10
$-2^-$	$-\infty$
$-2^+$	$+\infty$
$2^-$	0
$2^+$	0
$4^-$	3
$4^+$	3
$10^-$	4
$10^+$	4