P.3

25. $f(x) = \sqrt{9-x^2}$

$(x-h)^2 + (y-k)^2 = r^2$

center (h,k) ; radius r

$x^2 + y^2 = 9$

$y^2 = 9 - x^2$

$y = \pm\sqrt{9-x^2}$

$\cancel{(3-x)^2 \neq 9 - x^2}$

$(3-x)(3+x)$

1. $\frac{6x^5}{3x^5} \approx 2$ $y=2$

2. $\frac{7x^3}{3x^5} \approx \frac{3}{x^2} \to 0$ $y=0$

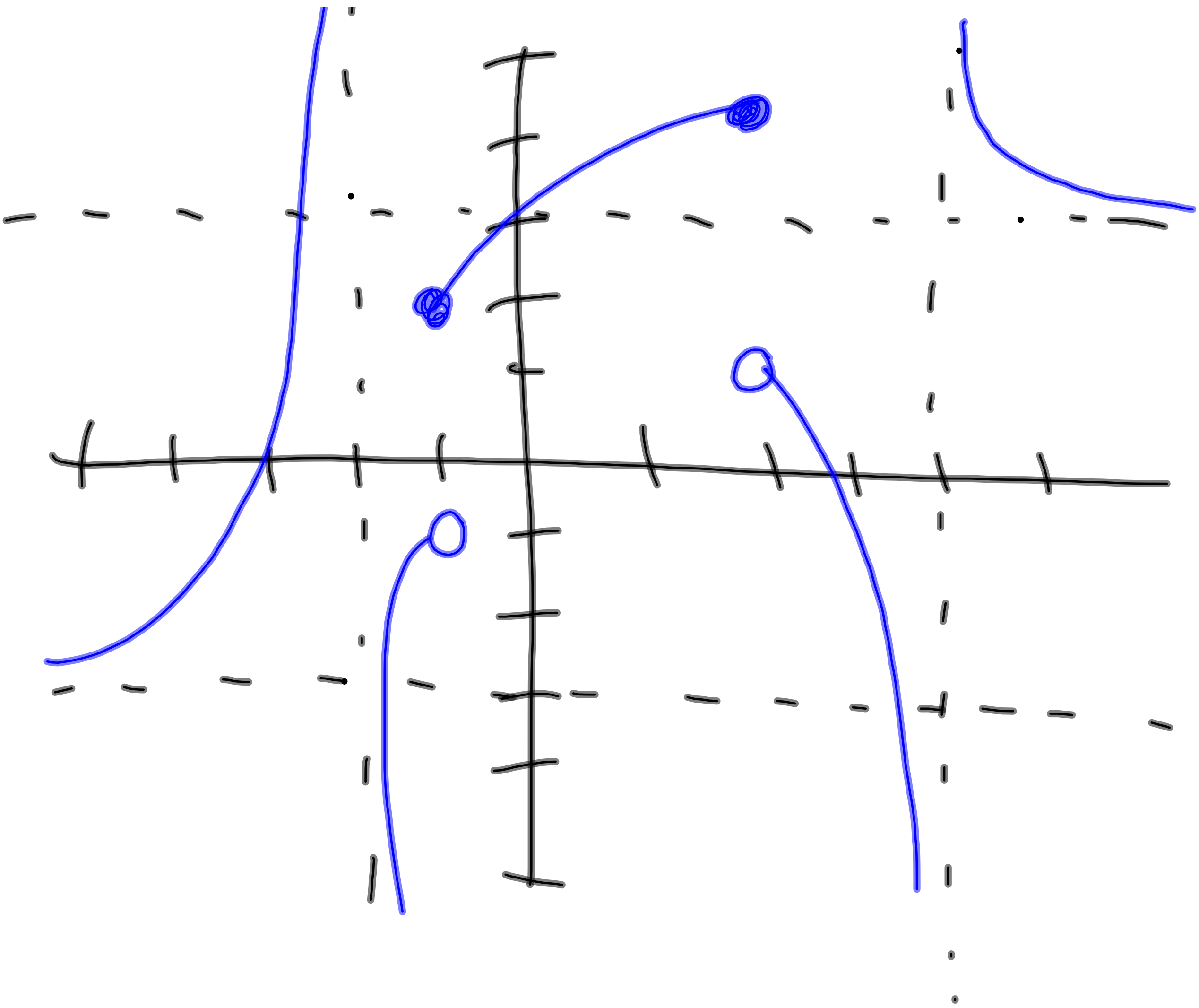
$$f(x) = \frac{(x-3)(x+3)}{(x-1)(x+1)}$$

zeros: 3, −3

x-int: (3,0) & (−3,0)

V.A.: $x=1$, $x=-1$

y-int: $\frac{0-9}{0-1} = 9$ $(0,9)$

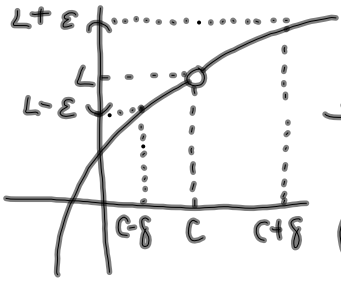


Informal Definition

If $f(x)$ becomes arbitrarily close to a single number L as x approaches c from either side, we say that the limit of $f(x)$ as x approaches c is L.

$$\lim_{x \to c} f(x) = L.$$

ε-δ Definition of the Limit



"$f(x)$ becomes arbitrarily close to L"
$\to$ $f(x)$ lies in the interval $(L-\varepsilon, L+\varepsilon)$ for some (really small) $\varepsilon > 0$.

$$|f(x) - L| < \varepsilon$$

"the distance between $f(x)$ and L is less than ε"

"x appoaches c" $\Rightarrow$ there exists a positive # δ such that x is either in the interval $(c-\delta, c)$ or $(c, c+\delta)$.

$$0 < |x - c| < \delta.$$

$x \neq c$

ε-δ Def: Let f be a function defined on an open interval containing c (except possibly at c) and let L be a real number.

The statement $\lim_{x \to c} f(x) = L$ means that for each $\varepsilon > 0$, there exists a $\delta > 0$ such that if $0 < |x - c| < \delta$, then $|f(x) - L| < \varepsilon$.

1.2

24. Find the limit L. Then, find $\delta > 0$ such that $|f(x) - L| < 0.01$ whenever $0 < |x - c| < \delta$.

$\varepsilon = 0.01$

$f(x) = 4 - \frac{x}{2}$

$c = 4$

$L = 2$

$$\lim_{x \to 4} \left(4 - \frac{x}{2}\right) = 2$$

$|f(x) - L| =$

$$\left|4 - \frac{x}{2} - 2\right| = \left|2 - \frac{x}{2}\right| = \left|\left(-\frac{1}{2}\right)(x-4)\right|$$

$$= \underbrace{\frac{1}{2}|x-4| < 0.01}_{} \Rightarrow |x-4| < 0.02$$

Let $\delta = 0.02$

read pp 52-54