ε - δ Definition of the Limit

Given $\varepsilon > 0$, $\exists\ \delta > 0$ s.t. $|f(x) - L| < \varepsilon$ whenever $|x - c| < \delta$.

$\lim_{x \to c} f(x) = L$ if

Find δ for $\varepsilon = 0.01$

26.

$$\lim_{x\to 5}(x^2+4) = 29$$

$f(x) = x^2+4$
$c = 5$
$L = 29$
$\varepsilon = 0.01$
$\delta = ?$

$$|f(x)-L| = |x^2+4-29|$$

$$= |x^2-25| = |(x+5)(x-5)| \le |(6+5)(x-5)|$$

$$= 11|x-5| < \varepsilon$$

$$\Rightarrow |x-5| < \varepsilon/11$$

call that δ

For $\boxed{\delta = \varepsilon/11}$, $|x-c|<\delta$ $(|x-5|<\varepsilon/11)$ guarantees that $|f(x)-L|<\varepsilon$

✱ General Strategy: manipulate $|f(x)-L|$ until it looks like a constant times $|x-c|$

28. Prove that the limit is L using ε-δ def.

$f(x) = 2x+5$

$$\lim_{x \to -3} (2x+5) = -1$$

$c = -3$

$L = -1$

Given $\varepsilon > 0$.

$$|f(x) - L| = |2x+5-(-1)| = |2x+6|$$

$$= 2|x+3| = 2|x-(-3)| < \varepsilon \iff |x-(-3)| < \varepsilon/2.$$

Take $\delta = \varepsilon/2$.

whenever $|x-c| = |x-(-3)| < \delta$,

$$|f(x)-L| = |2x+5-(-1)| = 2|x-(-3)| < 2\delta$$

$= 2 \cdot \varepsilon/2 = \varepsilon$. Therefore $\lim_{x\to -3} 2x+5 = -1$.

$$\lim_{x\to 7} 4x+2 = 30$$

Find L & prove limit is L using ε-δ.

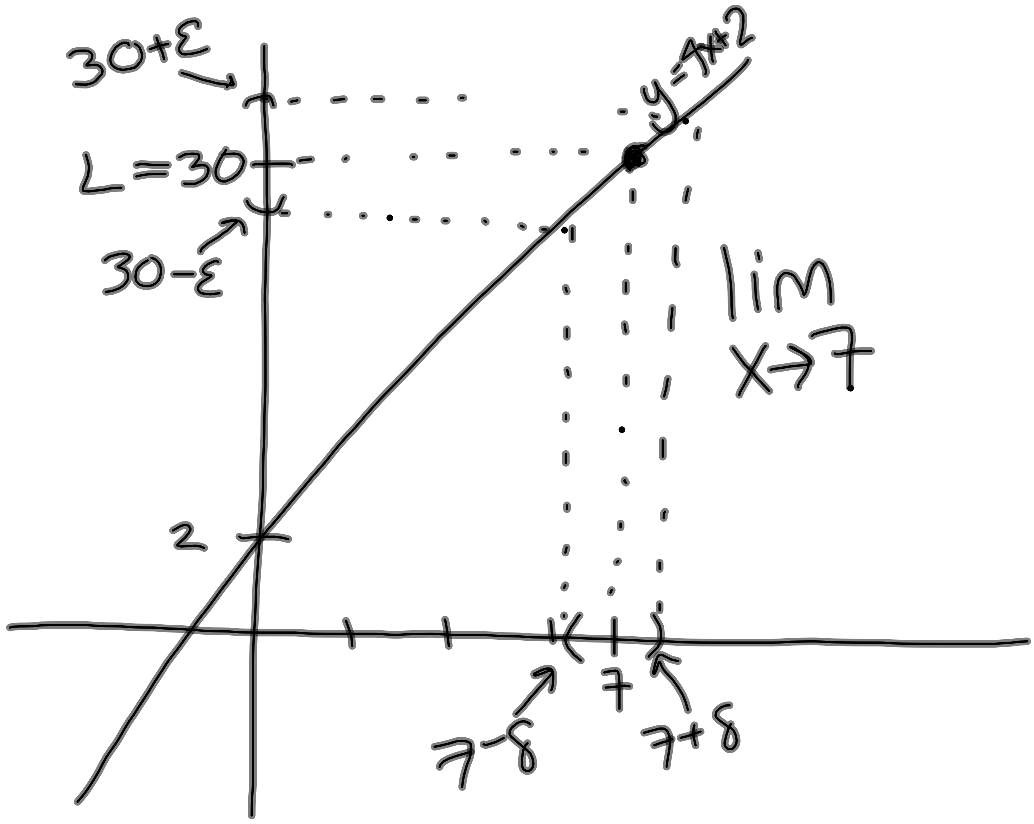
$$|f(x)-L| = |4x+2-30| = |4x-28| = 4|x-7| < \varepsilon$$

$$\Leftrightarrow |x-7| < \varepsilon/4$$

Given $\varepsilon > 0$, take $\delta = \varepsilon/4$. Then whenever $\underbrace{|x-7|<\delta}_{|x-c|<\delta}$, we have

$$\underbrace{|4x+2-30|}_{|f(x)-L|} = 4|x-7| \underbrace{<}_{<} 4\cdot\underbrace{\varepsilon/4}_{\delta} = \underbrace{\varepsilon}_{\varepsilon}.$$

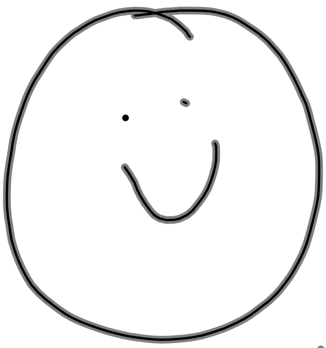
Therefore $\lim_{x\to 7} 4x+2 = 30$ by ε-δ definition.


30+ε
L=30
30−ε
y=4x+2
lim
x→7
2
7
7−δ
7+δ

1.2

$$\left.\begin{array}{l} 23, 24 \\ 27-30 \end{array}\right\} \varepsilon\text{-}\delta$$

1.3 Evaluating Limits Analytically



30 mins of Khan Academy this week.