$\underline{1.2}$

24. $\lim_{x\to 4}\left(4-\frac{x}{2}\right)=2$

$\varepsilon=0.01$

$\boxed{L=2}$

$f(x)=4-\frac{x}{2}$

$c=4$

$$|f(x)-L|=\left|4-\frac{x}{2}-2\right|=\left|2-\frac{x}{2}\right|$$

$$=\left|\left(-\frac{1}{2}\right)(x-4)\right|=\frac{1}{2}|x-4|<\varepsilon$$

$$\Longleftrightarrow |x-4|<2\varepsilon=\delta$$

$\boxed{\delta=0.02}$

30 $\lim_{x \to 1} \left(\frac{2}{3}x + 9 \right) = \frac{29}{3}$

$$\left| f(x) - L \right| = \left| \frac{2}{3}x + 9 - \frac{29}{3} \right| = \left| \frac{2}{3}x - \frac{2}{3} \right|$$

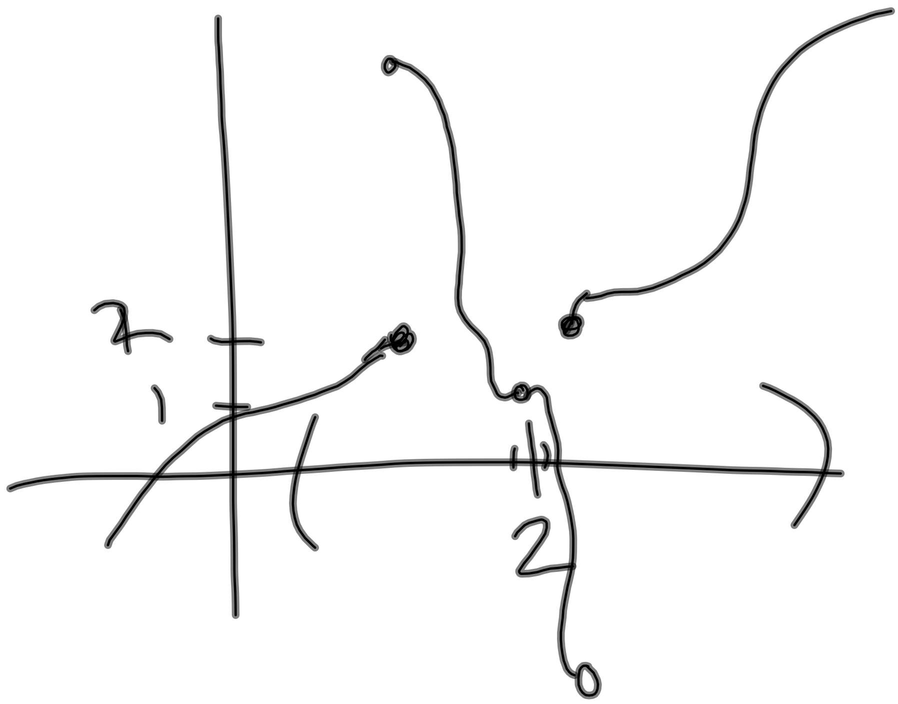
$$= \frac{2}{3} \left| x - 1 \right| < \varepsilon \iff \left| x - 1 \right| < \frac{3\varepsilon}{2}$$

Given $\varepsilon > 0$, we want to find $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $|x - c| < \delta$. Take $\delta = \frac{3\varepsilon}{2}$. Then

$$\left| f(x) - L \right| = \left| \frac{2}{3}x + 9 - \frac{29}{3} \right| = \frac{2}{3} \left| x - 1 \right| < \frac{2}{3} \cdot \frac{3\varepsilon}{2}$$

$= \varepsilon$ whenever $|x - c| = |x - 1| < \delta = \frac{3\varepsilon}{2}$.

Thus, $\lim_{x \to 1} \left(\frac{2}{3}x + 9 \right) = \frac{29}{3}$.


2
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2

1.3 Evaluating Limits Analytically

$\lim_{x \to c} f(x)$ does not depend on the value of $f(c)$, and this value does not have to be defined.

If $\lim_{x \to c} f(x) = f(c)$, we call the function <u>continuous</u> at c.

Theorem 1.1 Some Basic Limits

$b, c \in \mathbb{R}$; $n > 0$ an integer

1. $\lim_{x \to c} b = b$

2. $\lim_{x \to c} x = c$

3. $\lim_{x \to c} x^n = c^n$

Examples:

$\lim_{x \to 3} (-5) = -5$

$\lim_{x \to 4} x = 4$

$\lim_{x \to -2} x^3 = (-2)^3 = -8$

Theorem 1.2 more properties of Limits

$b, c \in \mathbb{R}$, $n > 0$ an integer, f & g - functions

$$\lim_{x \to c} f(x) = L \ ; \ \lim_{x \to c} g(x) = K$$

1. scalar multiple
$$\lim_{x \to c} [b f(x)] = bL$$

2. sum or difference
$$\lim_{x \to c} [f(x) \pm g(x)] = L \pm K$$

3. product
$$\lim_{x \to c} [f(x) g(x)] = LK$$

4. quotient
$$\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{K}, \ K \neq 0$$

5. power
$$\lim_{x \to c} [f(x)]^n = L^n$$ (follows from #3)

polynomials, rational functions, $\sqrt[n]{x}$, $f(g(x))$, sin, cos, etc.

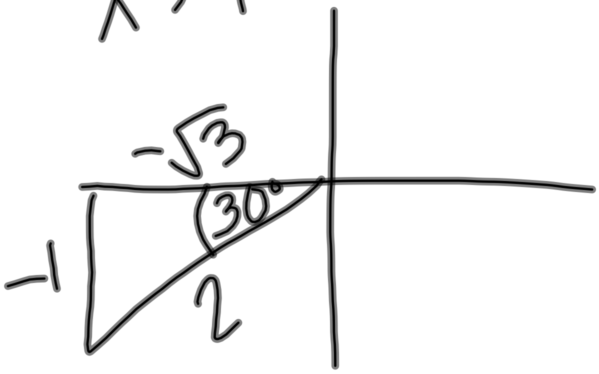
1.3

6. $\lim_{x \to -2} x^3 = (-2)^3 = \boxed{-8}$

18. $\lim_{x \to 3} \frac{\sqrt{x+1}}{x-4} = \frac{\sqrt{3+1}}{3-4} = \frac{\sqrt{4}}{-1} = \boxed{-2}$

30. $\lim_{x \to 1} \sin \frac{\pi x}{2} = \boxed{1}$

36. $\lim_{x \to 7} \sec\left(\frac{\pi x}{6}\right) = \sec \frac{7\pi}{6}$ -



38.

$$\lim_{x\to c} f(x) = \frac{3}{2} \; ; \; \lim_{x\to c} g(x) = \frac{1}{2}$$

(a) $\lim_{x\to c} [4f(x)] = 4 \cdot \frac{3}{2} = \boxed{6}$

(b) $\lim_{x\to c} [f(x)+g(x)] = \frac{3}{2} + \frac{1}{2} = \boxed{2}$

(c) $\lim_{x\to c} [f(x)g(x)] = \frac{3}{2} \cdot \frac{1}{2} = \boxed{\frac{3}{4}}$

(d) $\lim_{x\to c} \left[\frac{f(x)}{g(x)}\right] = \frac{3}{2} / \frac{1}{2} = \frac{3}{2} \cdot \frac{2}{1} = \boxed{3}$

42. $h(x) = \dfrac{x^2 - 3x}{x}$

(a) $\lim_{x \to -2} h(x) = \dfrac{(-2)^2 - 3(-2)}{-2} = \dfrac{4+6}{-2} = \boxed{-5}$

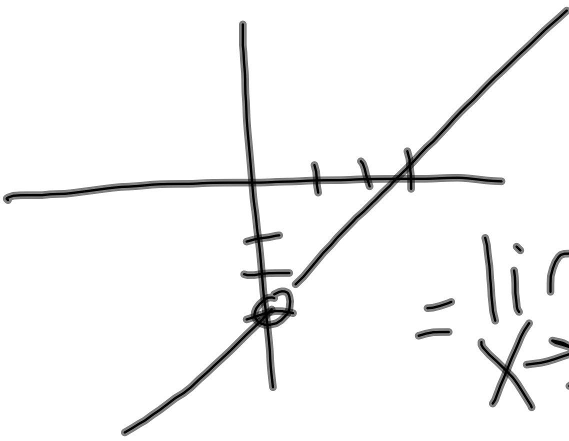
(b) $= \lim_{x \to 0} h(x) = \dfrac{0^2 - 3(0)}{0} = \dfrac{0}{0}$

substitution yields an indeterminate form.

we need to rewrite the expression.

$$\lim_{x \to 0} \frac{x^2 - 3x}{x} = \lim_{x \to 0} \frac{x(x-3)}{x}$$

$\dfrac{x(x-3)}{x} = x - 3$ everywhere except @ $x = 0$



$$\lim_{x \to 0} \frac{x(x-3)}{x} = $$

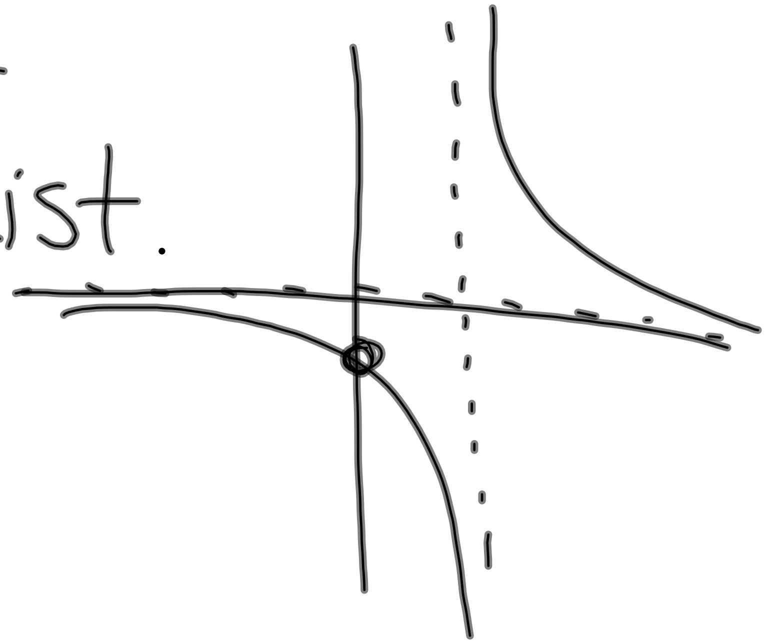
$$= \lim_{x \to 0} (x-3) = 0 - 3$$

$$= \boxed{-3}$$

44. $\lim_{x\to 1} \frac{x}{x^2-x} = \lim_{x\to 1} \frac{x}{x(x-1)}$

$= \lim_{x\to 1} \frac{1}{x-1}$

does not exist.



48. $$\lim_{x\to -1} \frac{x^3+1}{x+1} = \lim_{x\to -1} \frac{\cancel{(x+1)}(x^2-x+1)}{\cancel{x+1}}$$

$$a^3+b^3=(a+b)(a^2-ab+b^2)$$
$$a^3-b^3=(a-b)(a^2+ab+b^2)$$

$$= \lim_{x\to -1} (x^2-x+1) = (-1)^2-(-1)+1$$

$$= 1+1+1 = \boxed{3}$$

$$54.\ \lim_{x\to 0}\left(\frac{\sqrt{2+x}-\sqrt{2}}{x}\right)\cdot\left(\frac{\sqrt{2+x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{2}}\right)$$

$$=\lim_{x\to 0}\frac{2+x-2}{x\left(\sqrt{2+x}+\sqrt{2}\right)}=\lim_{x\to 0}\frac{\cancel{x}}{\cancel{x}\left(\sqrt{2+x}+\sqrt{2}\right)}$$

$$=\lim_{x\to 0}\frac{1}{\sqrt{2+x}+\sqrt{2}}=\frac{1}{2\sqrt{2}}=\boxed{\frac{\sqrt{2}}{4}}$$

page 65!