56: $\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3} \cdot \frac{\sqrt{x+1}+2}{\sqrt{x+1}+2}$

$= \lim_{x\to 3} \frac{x+1-4}{(x-3)(\sqrt{x+1}+2)} = \lim_{x\to 3} \frac{\cancel{x-3}}{\cancel{(x-3)}(\sqrt{x+1}+2)}$

$= \lim_{x\to 3} \frac{1}{\sqrt{x+1}+2} = \frac{1}{\sqrt{3+1}+2} = \boxed{\frac{1}{4}}$

58. $\lim_{x\to 0} \frac{\frac{4}{4}\cdot\frac{1}{x+4} - \frac{1}{4}\cdot\frac{x+4}{x+4}}{x}$

$= \lim_{x\to 0} \frac{\frac{4}{4(x+4)} - \frac{x+4}{4(x+4)}}{\frac{x}{1}} = \lim_{x\to 0} \frac{4-x-4}{4(x+4)} \cdot \frac{1}{x}$

$= \lim_{x\to 0} \frac{-\cancel{x}}{4(x+4)} \cdot \frac{1}{\cancel{x}} = \lim_{x\to 0} \frac{-1}{4(x+4)} = \boxed{\frac{-1}{16}}$

62. $\lim\limits_{\Delta x \to 0} \dfrac{(x+\Delta x)^3 - x^3}{\Delta x}$

$= \lim\limits_{\Delta x \to 0} \dfrac{(\cancel{x^3} + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3) - \cancel{x^3}}{\Delta x}$

$= \lim\limits_{\Delta x \to 0} \dfrac{\cancel{\Delta x}(3x^2 + 3x\Delta x + (\Delta x)^2)}{\cancel{\Delta x}}$

$= \lim\limits_{\Delta x \to 0} 3x^2 + 3x\Delta x + (\Delta x)^2$

$= \boxed{3x^2}$

$(a+b)^3 \stackrel{?}{=} a^3 + 3a^2b + 3ab^2 + b^3$

64. $\lim_{x\to16} \frac{4-\sqrt{x}}{x-16} \cdot \frac{4+\sqrt{x}}{4+\sqrt{x}} =$

$\lim_{x\to16} \frac{\cancel{16-x}^{\,-1}}{\cancel{(x-16)}(4+\sqrt{x})} = \lim_{x\to16} \frac{-1}{4+\sqrt{x}} =$

$= \boxed{-\frac{1}{8}}$

66. $\lim\limits_{x\to 2} \dfrac{x^5-32}{x-2}$

$$
\begin{array}{r|rrrrrr}
2 & 1 & 0 & 0 & 0 & 0 & -32 \\
 & & 2 & 4 & 8 & 16 & 32 \\
\hline
 & 1 & 2 & 4 & 8 & 16 & \boxed{0} \\
 & x^4 & x^3 & x^2 & x & c &
\end{array}
$$

$$= \lim_{x\to 2} \frac{\cancel{(x-2)}(x^4+2x^3+4x^2+8x+16)}{\cancel{x-2}}$$

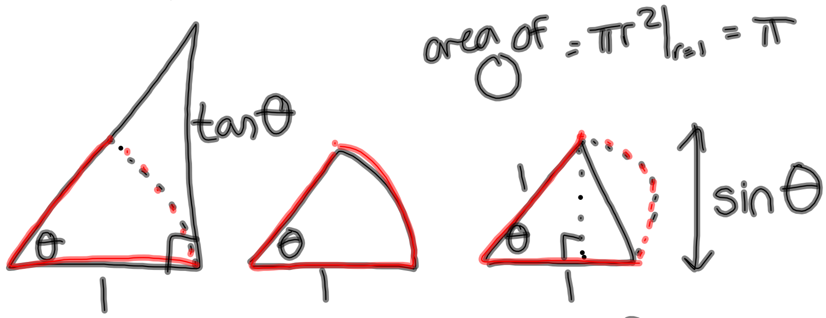
$$= 2^4+2(2)^3+4(2^2)+8(2)+16 = \boxed{80}$$

$$= 16+16+16+16+16 =$$

Squeeze Theorem

$$\lim_{x\to 0} \frac{\sin x}{x} = ?$$

area of ◯ $= \pi r^2 \big|_{r=1} = \pi$



area of triangle $\geq$ area of sector $\geq$ area of triangle

$$\frac{\tan\theta}{2} \geq \frac{\theta}{2} \geq \frac{\sin\theta}{2}$$

$\frac{\sin\theta}{2\cos\theta}$ ''

$$\frac{\cancel{\text{area of whole circle}}\ \pi}{\cancel{2\pi}} = \frac{\text{area of sector}}{\theta}$$

$$\frac{1}{2} = \frac{\text{area of sector}}{\theta}$$

multiply through by $\frac{2}{\sin\theta}$

$$\frac{1}{\cos\theta} \geq \frac{\theta}{\sin\theta} \geq 1$$

take reciprocals & reverse inequalities

$$\cos\theta \leq \frac{\sin\theta}{\theta} \leq 1$$

$$\lim_{\theta\to 0} \cos\theta \leq \lim_{\theta\to 0} \frac{\sin\theta}{\theta} \leq \lim_{\theta\to 0} 1$$

$$1 \leq \lim_{\theta\to 0} \frac{\sin\theta}{\theta} \leq 1$$

$$\Rightarrow \lim_{\theta\to 0} \frac{\sin\theta}{\theta} = \boxed{1}$$

If $f(x) \le g(x) \le h(x)$ and

$$\lim_{x \to c} f(x) = L = \lim_{x \to c} h(x),$$

then $\lim_{x \to c} g(x) = L$.

$$\lim_{x \to 0} \frac{\sin x}{x} = 1 \ ; \ \lim_{x \to 0} \frac{1 - \cos x}{x} = 0$$

☆ Memorize these Limits!

use squeeze Thm to find

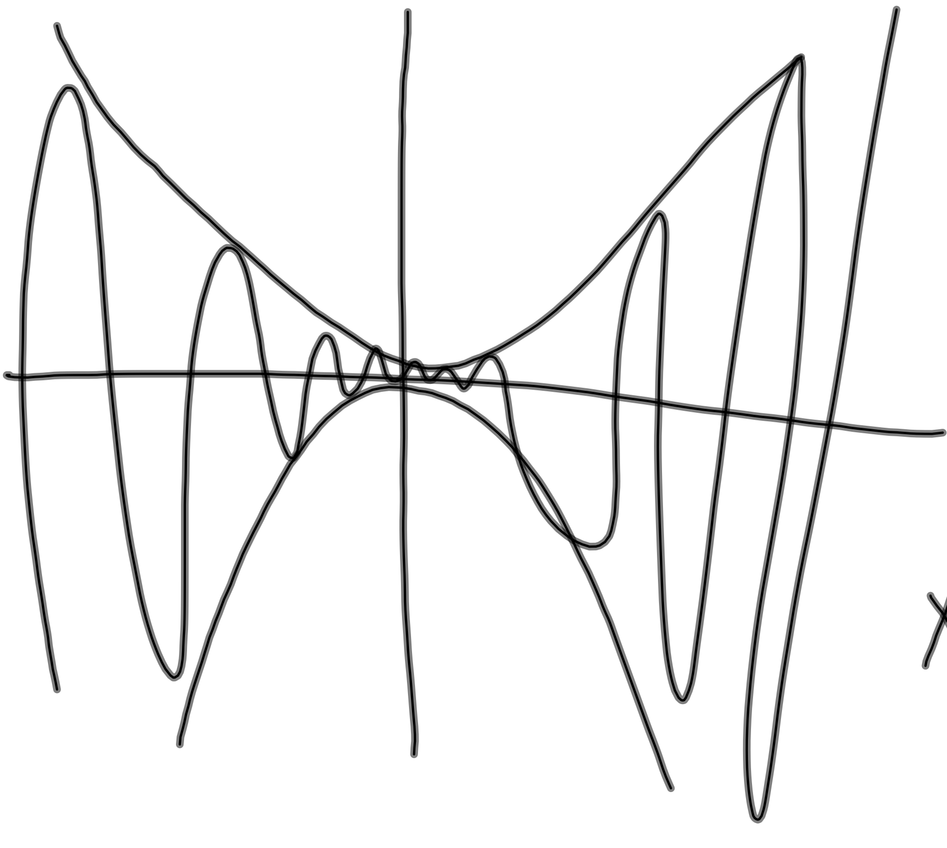
$$\lim_{x \to 0} x^2 \cos x$$

$$-1 \le \cos x \le 1$$

$$-x^2 \le x^2 \cos x \le x^2$$

$$\underbrace{\lim_{x \to 0} (-x^2)}_{=0} \le \lim_{x \to 0} x^2 \cos x \le \underbrace{\lim_{x \to 0} x^2}_{=0}$$

$$\Rightarrow \lim_{x \to 0} x^2 \cos x = \boxed{0}$$



$x^2 \sin x$

68. $\lim_{x \to 0} \frac{3(1-\cos x)}{x}$

$= \lim_{x \to 0} (3) \cdot \lim_{x \to 0} \frac{(1-\cos x)}{x}$

$= 3 \cdot 0$

$= \boxed{0}$

72. $$\lim_{x\to 0} \frac{\tan^2 x}{x} = \lim_{x\to 0} \frac{\sin^2 x}{x\cos^2 x}$$

$$= \lim_{x\to 0} \frac{\sin x}{x} \cdot \lim_{x\to 0} \frac{\sin x}{\cos^2 x}$$

$$= 1 \cdot 0$$

$$= \boxed{0}$$

78. $$\lim_{x\to 0} \frac{\sin 2x}{\sin 3x}$$

$$= \lim_{x\to 0} \frac{\sin 2x}{2x} \cdot \frac{3x}{\sin 3x} \cdot \frac{2}{3}$$

$$= \lim_{x\to 0} \frac{\sin 2x}{2x} \cdot \lim_{x\to 0} \frac{3x}{\sin 3x} \cdot \lim_{x\to 0} \frac{2}{3}$$

$$1 \cdot 1 \cdot 2/3$$

$$= \boxed{2/3}$$

as $x \to 0$

$$\frac{\sin x}{x} \to 1$$

$$\frac{\sin 2x}{2x} \to 1$$

$$\frac{\sin 3x}{3x} \to 1$$

HW is page 66!

Quiz tomorrow!