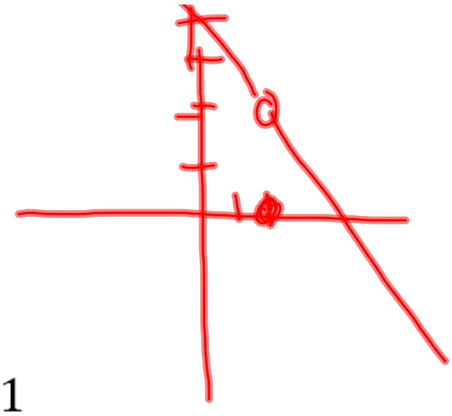
Review - Find the limits (if they exist):

1. $\lim\limits_{x\to 2} f(x)$, where $f(x) = \begin{cases} 4-x, & x \neq 2 \\ 0, & x = 2 \end{cases}$

$= \boxed{2\ !}$



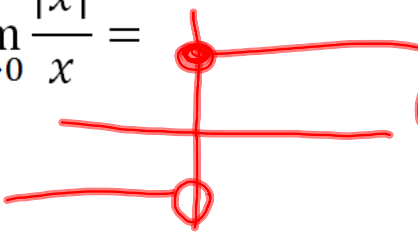
2. $\lim\limits_{x\to 1} f(x)$, where $f(x) = \begin{cases} 4x-7, & x \neq 1 \\ 5, & x = 1 \end{cases}$

$= \boxed{-3}$

$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

$\frac{|x|}{x} = \begin{cases} \frac{x}{x} = 1, & x \geq 0 \\ \frac{-x}{x} = -1, & x < 0 \end{cases}$

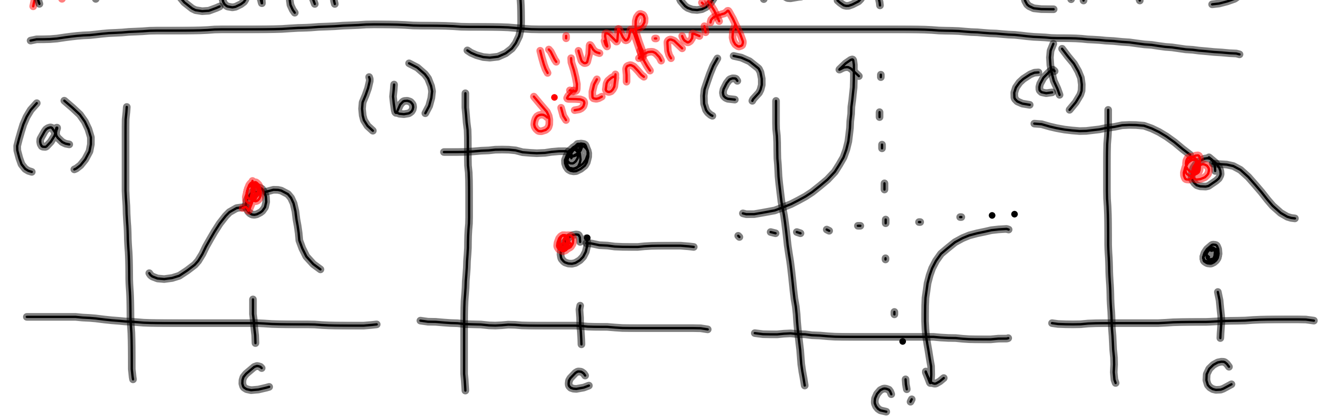
3. $\lim\limits_{x\to 0} \frac{|x|}{x} =$



does not exist

4. $\lim\limits_{x\to 0} \frac{1-\cos x}{x} = 0$

# 1.4 Continuity and One-Sided Limits




(a) $f(c)$ is undefined

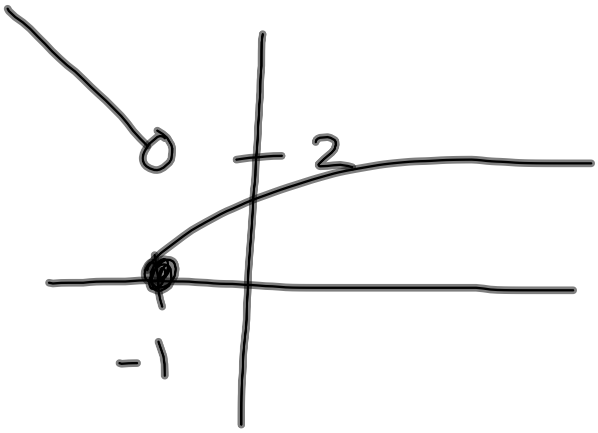
(b), (c) $\lim_{x \to c} f(x)$ does not exist

(d) $\lim_{x \to c} f(x) \neq f(c)$

these are all discontinuities

(a) & (d) are removable

(b) & (c) are nonremovable




$\lim_{x \to -1^-} f(x) = 2$

$\lim_{x \to -1^+} f(x) = 0$

$\lim_{x \to -1} f(x)$ does not exist

## one-sided limits

$\lim_{x \to c^+} f(x)$ = limit from the right

$\lim_{x \to c^-} f(x)$ = limit from the left

$\lim_{x \to c} f(x) = L$ if and only if $\lim_{x \to c^-} f(x) = L = \lim_{x \to c^+} f(x)$

# Definition of Continuity

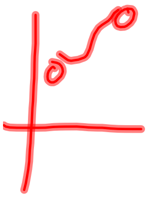
A function $f$ is <u>continuous at $c$</u>

if : (1) $f(c)$ is defined

(2) $\lim\limits_{x \to c} f(x)$ exists

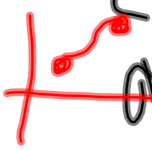
(3) $\lim\limits_{x \to c} f(x) = f(c)$

$f$ is <u>continuous on an open interval</u> $(a, c)$ if it is continuous at each point on the interval



* if it is continuous on $(-\infty, \infty)$, we say it is <u>everywhere continuous</u>

$f$ is <u>continuous on a closed interval</u> $[a, b]$ if it is continuous on $(a, b)$



and $\lim\limits_{x \to a^+} f(x) = f(a)$ and $\lim\limits_{x \to b^-} f(x) = f(b)$.

$$10.\ \lim_{x\to 4^-} \frac{\sqrt{x}-2}{x-4}\cdot\frac{\sqrt{x}+2}{\sqrt{x}+2}=$$

$$\lim_{x\to 4^-} \frac{\cancel{x-4}}{\cancel{(x-4)}(\sqrt{x}+2)} = \boxed{\frac{1}{4}}$$

$$12.\ \lim_{x\to 2^+} \frac{|x-2|}{x-2} = \boxed{1}$$

$x-2\geq 0$

$$\frac{|x-2|}{x-2} = \begin{cases} \frac{x-2}{x-2}=1, & x\geq 2 \\ \frac{-(x-2)}{x-2}=-1, & x<2 \end{cases}$$

$$|x| = \begin{cases} x, & x\geq 0 \\ -x, & x<0 \end{cases}$$