Describe the dis/continuity of the function.

$$f(x) = \frac{x+2}{x^2 - 3x - 10} = \frac{(x+2)(x+5)(x-2)}{(x-5)(x+2)(x+3)(x+5)}$$

f has removable discontinuities @ $x=-2, -5$

f has non-removable discontinuities @ $x=5, -3$

f is continuous on: $(-\infty,-2)\cup(-2,5)\cup(5,\infty)$

$$f(x) = \begin{cases} -2x, & x \le 2 \\ x^2 - 4x + 1, & x > 2 \end{cases}$$

$-2(2) = -4$

$2^2 - 4(2) + 1 = 4 - 8 + 1 = -3$

f has a non-removable discontinuity @ $x=2$.

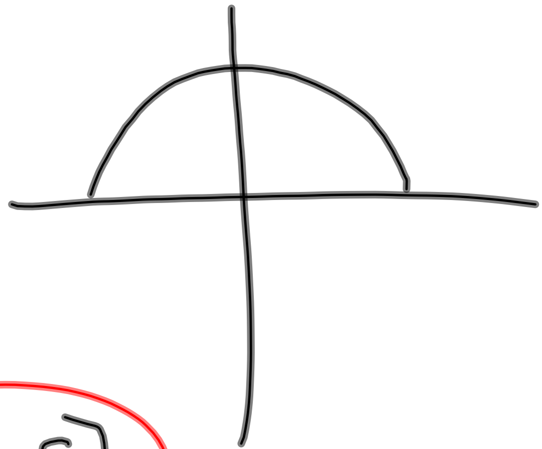
f is continuous on $(-\infty,2)\cup(2,\infty)$

29. $g(x) = \sqrt{25 - x^2}$ $\quad [-5, 5]$

$x^2 + y^2 = 5^2$

$y^2 = 25 - x^2$

$y = \pm\sqrt{25 - x^2}$



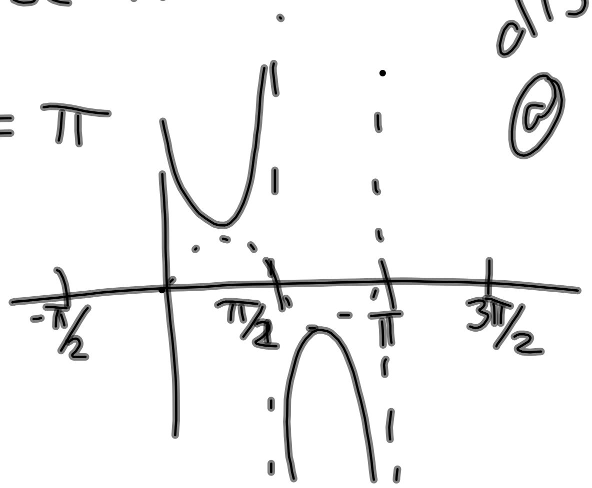
cts on $(-5, 5]$

61. $f(g(x)) = (x-1)^2$

cts on $(-\infty, \infty)$

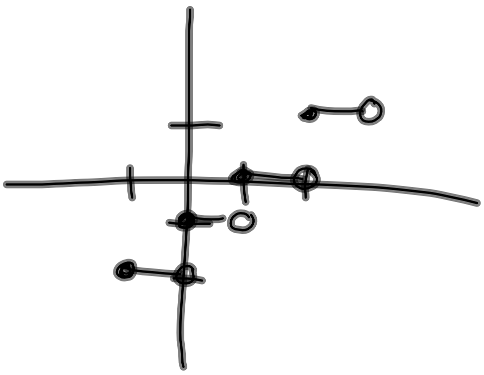
51. $f(x) = \csc 2x$

period: $\frac{2\pi}{2} = \pi$



non-removable discontinuities @ $\frac{k\pi}{2}$, $k \in \mathbb{Z}$

53. $f(x) = [\![x-1]\!]$

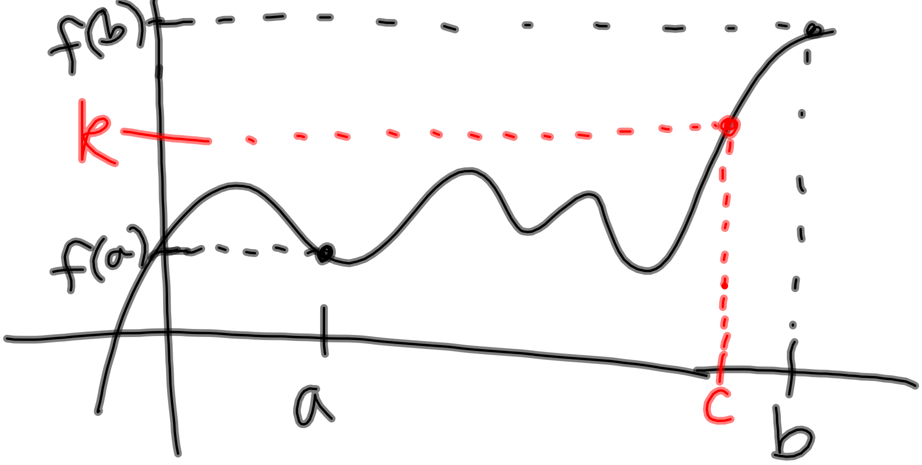


nonremovable discontinuities at all integers

1.4 cont.

Intermediate Value Theorem

If f is continuous on the closed interval $[a,b]$ and k is any number between $f(a)$ and $f(b)$, then there is at least one number c in $[a,b]$ such that $f(c)=k$.



IVT can guarantee zeros!

(if $f(a)$ & $f(b)$ are opposite signs)

76. $f(x)=x^3+3x-2$, $[0,1]$

does IVT guarantee that f has any zeros in this interval?

$f(0) = -2$

$f(1) = 1+3-2 = 2$

yes, IVT guarantees a zero

78. $f(x) = -\frac{4}{x} + \tan\frac{\pi x}{8}$, $[1,3]$

$f(1) = -4 + \tan\frac{\pi}{8} < 0$
≈ -3.6

$f(3) = -\frac{4}{3} + \tan\frac{3\pi}{8} > 0$
≈ 1.08

, yes, IVT guarantees a zero

84. $f(x) = x^2 - 6x + 8$, $[0,3]$

does IVT guarantee a zero?

$f(0) = 8$

$f(3) = 3^2 - 6(3) + 8 = -1$

yes

If yes, what is it?

$x^2 - 6x + 8 = 0$

$(x-4)(x-2) = 0$

$x = \boxed{2}, \cancel{4}$

86. $f(x) = \dfrac{x^2+x}{x-1}$, $\left[\frac{5}{2}, 4\right]$, $f(c) = 6$.

does IVT guarantee a $c \in \left[5/2, 4\right]$ s.t. $f(c) = 6$?

$$f(5/2) = \frac{(5/2)^2 + \frac{5}{2}}{\frac{5}{2} - 1} = \frac{\frac{25}{4} + \frac{10}{4}}{\frac{5}{2} - \frac{2}{2}} = \frac{\frac{35}{4}}{\frac{3}{2}} = \frac{35}{\cancel{4}_2} \cdot \frac{\cancel{2}}{3} = \frac{35}{6} < 6$$

$$f(4) = \frac{4^2+4}{4-1} = \frac{20}{3} > 6$$

Yes, IVT guarantees such a c.

$$\frac{x^2+x}{x-1} = 6$$

$$x^2 + x = 6(x-1)$$

$$x^2 + x = 6x - 6$$

$$x^2 - 5x + 6 = 0$$

$$(x-3)(x-2) = 0$$

$$x = \cancel{2}, \boxed{3}$$

IVT problems

1.4 #75-86

Test is Wed. 9/12

1. 2
2. 2
3. 6
4. 2
5. $-\infty$
6. 1
7. DNE
8. $1/6$
9. -2
10. -6

A. $f(c)$
B. L