Evaluate the limits:

1. $$\lim_{x\to 0}\frac{\sin x}{x} = 1$$

2. $$\lim_{x\to 0}\frac{1-\cos x}{x} = 0$$

$$\lim_{x\to 4}\frac{\sqrt{x+5}-3}{x-4} \cdot \frac{\sqrt{x+5}+3}{\sqrt{x+5}+3} = \lim_{x\to 4}\frac{\overset{x-4}{\cancel{x+5-9}}}{\cancel{(x-4)}(\sqrt{x+5}+3)}$$

$$= \lim_{x\to 4}\frac{1}{\sqrt{x+5}+3} = \frac{1}{\sqrt{9}+3} = \boxed{\frac{1}{6}}$$

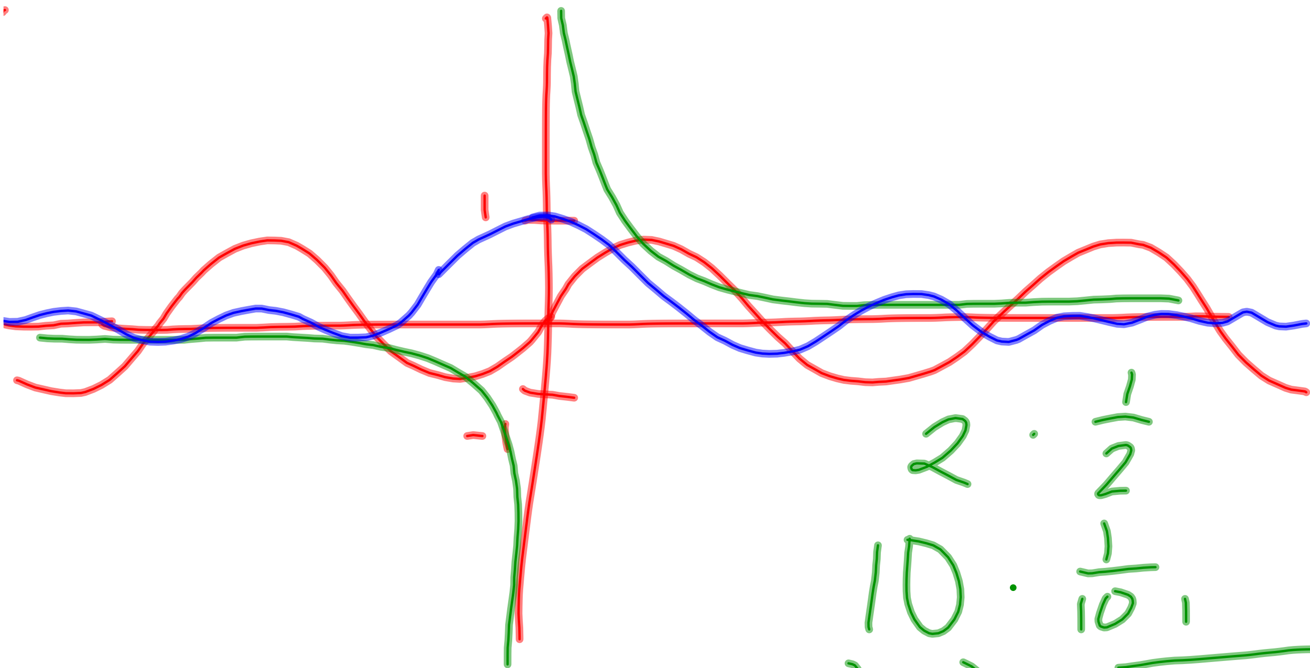
1.4

74. $f(x) = \frac{x^3-8}{x-2} = \frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{x-2}}$

$= x^2+2x+4$ everywhere except @ $x=2$

73. $f(x) = \frac{\sin x}{x}$ discontinuity @ $x=0$

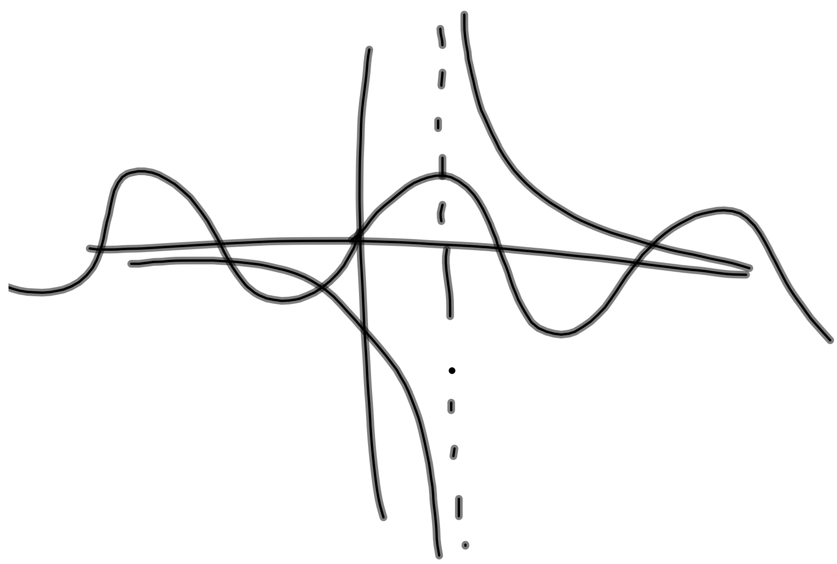
$= \sin x \cdot \frac{1}{x}$



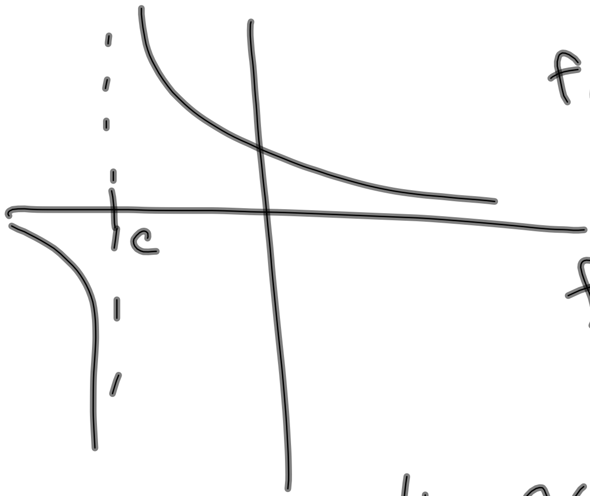

$2 \cdot \frac{1}{2}$

$10 \cdot \frac{1}{10}$, 1

$10000 \cdot \frac{1}{10000}$



# 1.5 Infinite Limits




$f(x) \to \infty$ as $x \to c^+$

$f(x) \to -\infty$ as $x \to c^-$

to say that $\lim\limits_{x \to c^-} f(x) = -\infty$

& $\lim\limits_{x \to c^+} f(x) = \infty$

means that the function increases/decreases without bound

(technically, the limit does not exist)

$$\lim_{x \to 0^-} \frac{1}{x}$$

(a) 0

(b) 5

(c) Does not exist

(d) none of the above

$\lim_{x \to 0} \frac{1}{x}$ does not exist

$\lim_{x \to 0} \frac{1}{x^2} = \infty$

$\lim_{x \to 0} \left(-\frac{1}{x^2}\right) = -\infty$

Theorem 1.15

If $\lim_{x \to c} f(x) = \infty$ & $\lim_{x \to c} g(x) = L$

where $c, L \in \mathbb{R}$

Then:

(1) $\lim_{x \to c} \left[ f(x) \pm g(x) \right] = \infty$

(2) $\lim_{x \to c} \left[ f(x) g(x) \right] = \begin{cases} \infty & \text{if } L > 0 \\ -\infty & \text{if } L < 0 \end{cases}$

(3) $\lim_{x \to c} \dfrac{g(x)}{f(x)} = 0$

14. $f(x) = \dfrac{-4x}{x^2+4}$

has no V.A.'s

28. $g(\theta) = \dfrac{\tan\theta}{\theta} = \dfrac{\sin\theta}{\cos\theta}\cdot\dfrac{1}{\theta} = \dfrac{\sin\theta}{\theta}\cdot\dfrac{1}{\cos\theta}$

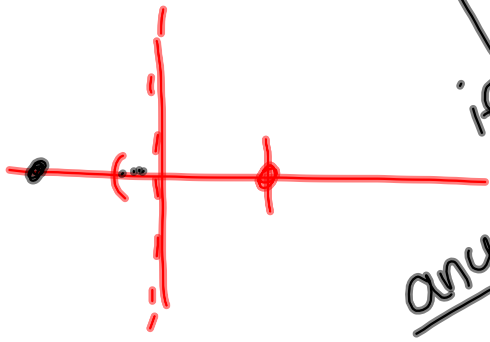
V.A.'s @ $\dfrac{k\pi}{2}$, k odd

24. $h(x) = \dfrac{\overset{\cancel{(x+2)}(x-2)}{x^2-4}}{\underset{\underset{\cancel{(x+2)}(x^2+1)}{x^2(x+2)+1(x+2)}}{x^3+2x^2+x+2}}$

no V.A.'s

42. $$\lim_{x\to 0^-}\left(\frac{x}{x}x^2-\frac{1}{x}\right) = \lim_{x\to 0^-}\left(\frac{\overset{(x-1)(x^2+x+1)}{x^3-1}}{x}\right) = \boxed{\infty}$$

$$\frac{x^3}{x}-\frac{1}{x}$$



if we plug in any negative #, $\frac{x^3-1}{x} > 0$

46. $$\lim_{x\to 0}\frac{x+2}{\cot x}$$

$$= \frac{\lim_{x\to 0} x+2}{\lim_{x\to 0}\cot x} = \frac{2}{\lim_{x\to 0}\cot x}$$

$$= \boxed{0}$$

48. $\lim\limits_{x\to\frac{1}{2}} x^2 \tan \pi x$

$= \left(\lim\limits_{x\to\frac{1}{2}} x^2\right)\left(\lim\limits_{x\to\frac{1}{2}} \tan(\pi x)\right)$

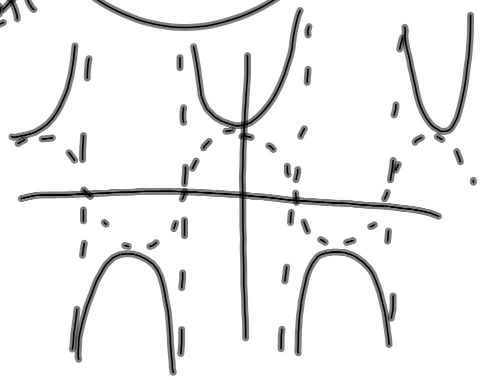
$= \left(\frac{1}{2}\right)^2 \lim\limits_{x\to\frac{1}{2}} \tan \pi x$

$\to \infty$ from right
$-\infty$ from left

does not exist

52. $\lim\limits_{x\to 3^+} \sec\frac{\pi x}{6}$

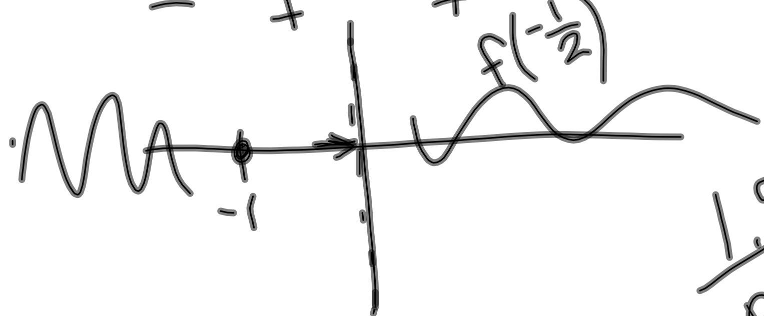
$= \boxed{-\infty}$



$\equiv \lim\limits_{x\to\frac{\pi}{2}^+} \sec x$

$f(x) = \dfrac{\overset{+}{(x+1)}\overset{-}{(x-3)}}{\underset{-}{x}\underset{+}{(x+3)}\underset{+}{(x+2)}}$ ; $\lim\limits_{x\to 0^-} f(x)$

$= \boxed{\infty}$




1.5
p85 odd