1.5

37. $$\lim_{x\to -3^-} \frac{x^2+2x-3}{x^2+x-6} = \frac{\cancel{(x+3)}(x-1)}{\cancel{(x+3)}(x-2)}$$

$$\lim_{x\to -3^-} \frac{x-1}{x-2} = \frac{-3-1}{-3-2} = \frac{-4}{-5} = \boxed{\frac{4}{5}}$$

45. $$\lim_{x\to \pi} \frac{\sqrt{x}}{\csc x} = \lim_{x\to \pi} \sqrt{x}\sin x$$

$$= \sqrt{\pi}\cdot \sin \pi = \boxed{0}$$

1. Find the limit , then use the $\varepsilon - \delta$ definition to prove that the limit is L .

$\lim_{x \to 8}(3x - 20) = 3(8) - 20 = 4$

$f(x) = 3x - 20$

$L = 4$

$c = 8$

Given $\varepsilon > 0$,

we want to find $\delta > 0$ s.t.

$|f(x) - L| < \varepsilon$ whenever $|x - c| < \delta$.

$|3x - 20 - 4| = |3x - 24| = 3|x - 8| < \varepsilon$

$\iff |x - 8| < \varepsilon/3$. Take $\delta = \varepsilon/3$.

2. Find the limit (if it exists).

$$\lim_{x\to-3}\frac{x^2+7x+12}{x^2-9} = \lim_{x\to-3}\frac{\cancel{(x+3)}(x+4)}{\cancel{(x+3)}(x-3)}$$

$$= \lim_{x\to-3}\frac{x+4}{x-3} = \frac{-3+4}{-3-3} = \boxed{-\frac{1}{6}}$$

3. Find the limit (if it exists).

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \quad , \quad where\ f(x) = 5x^2 + 3$$

$$\lim_{h\to 0} \frac{5(x+h)^2+3 - (5x^2+3)}{h} =$$

$$= \lim_{h\to 0} \frac{5(x^2+2xh+h^2)+\cancel{3} - 5x^2 - \cancel{3}}{h} = \lim_{h\to 0} \frac{\cancel{5x^2}+10xh+5h^2-\cancel{5x^2}}{h}$$

$$= \lim_{h\to 0} \frac{10xh+5h^2}{h} = \lim_{h\to 0} \frac{\cancel{h}(10x+5h)}{\cancel{h}} = \lim_{h\to 0} 10x+5h$$

$$= \boxed{10x}$$

4. Find the limit (if it exists).

$$\lim_{x \to 0} \frac{5 \sin 2x}{3x} =$$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\frac{\sin 2x}{2x} \to 1$$

$$= \lim_{x \to 0} \frac{\sin 2x}{2x} \cdot \frac{5 \cdot 2}{3}$$

$$= \underbrace{\lim_{x \to 0} \frac{\sin 2x}{2x}}_{1} \cdot \underbrace{\lim_{x \to 0} \frac{10}{3}}_{10/3} = \boxed{\frac{10}{3}}$$

5. Find the limit (if it exists).

$\lim_{x \to -5} f(x)$, $\quad f(x) = \begin{cases} -x^2 + 8, & x \leq -5 \\ 2x + 3, & x > -5 \end{cases}$

$\lim_{x \to -5^-} f(x) = -17$

$\lim_{x \to -5^+} f(x) = -7$

$-(-5)^2 + 8 = -25 + 8 = -17$

$\neq$

$2(-5) + 3 = -10 + 3 = -7$

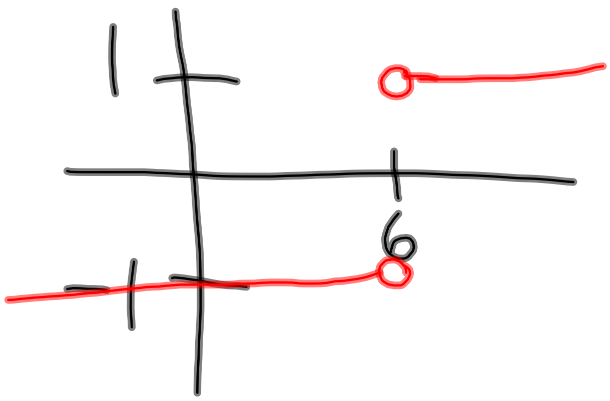
$\Rightarrow$ the limit does not exist

6. Find the limit (if it exists). Show SOME sort of work, whether it's a graph or definition of absolute value of x-6 -- show me how you arrived at your answer.

$$\lim_{x\to 6^-} \frac{|x-6|}{x-6} = \boxed{-1}$$

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$\frac{|x-6|}{x-6} = \begin{cases} \frac{x-6}{x-6} = 1, & x-6 > 0,\ x > 6 \\ \frac{-(x-6)}{x-6} = -1, & x-6 < 0,\ x < 6 \end{cases}$$



7. Use the Squeeze Theorem to find $\lim_{x\to 0} f(x)$. You <u>must</u> show use of the squeeze theorem.

$f(x) = 5x^2 \sin\frac{1}{x}$

$$-1 \le \sin\frac{1}{x} \le 1$$

$$-5x^2 \le 5x^2 \sin\frac{1}{x} \le 5x^2$$

$$\lim_{x\to 0}(-5x^2) \le \lim_{x\to 0} 5x^2 \sin\frac{1}{x} \le \lim_{x\to 0} 5x^2$$

$$\downarrow \qquad\qquad \downarrow$$

$$0 \qquad\qquad 0$$

$\therefore$ By Squeeze Thm,

$$\lim_{x\to 0} 5x^2 \sin\frac{1}{x} = 0.$$

8. Determine if the Intermediate Value Theorem guarantees a c in the interval $[-2, 3]$ such that $f(c) = -4$, and if so, find all such values of c.

$f(x) = x^2 - 7x + 2$

$f(-2) = (-2)^2 - 7(-2) + 2 = 4 + 14 + 2 > -4$

$f(3) = 3^2 - 7(3) + 2 = 9 - 21 + 2 < -4$

$\Rightarrow$ IVT does guarantee such a c.

$x^2 - 7x + 2 = -4$

$x^2 - 7x + 6 = 0$

$(x-6)(x-1) = 0$

$\cancel{x = 6}$, $x = 1$

9. Discuss the continuity of the function (identify all discontinuities, if any, as removable or non-removable).

$$f(x) = \frac{x^2 - 7x + 10}{x^2 - 3x + 2} = \frac{(x-5)(x-2)}{(x-2)(x-1)}$$

removable discontinuity @ $x=2$

non-removable discontinuity @ $x=1$

continuous on:

$$(-\infty, 1) \cup (1,2) \cup (2,\infty)$$

10. Find the limit (if it exists).

$$\lim_{x\to-2^+}\frac{\sqrt{x+11}-3}{x^2+5x+6}\cdot\frac{\sqrt{x+11}+3}{\sqrt{x+11}+3}$$

$$=\lim_{x\to-2^+}\frac{x+11-9}{(x^2+5x+6)(\sqrt{x+11}+3)}$$

$$=\lim_{x\to-2^+}\frac{\cancel{x+2}\ 1}{\cancel{(x+2)}(x+3)(\sqrt{x+11}+3)}=\frac{1}{(-2+3)(\sqrt{-2+11}+3)}$$

$$=\frac{1}{1(\sqrt{9}+3)}=\boxed{\frac{1}{6}}$$

$$\lim_{x\to 2^-} 3[\![x]\!]-5$$

$$= \lim_{x\to 2^-} 3[\![x]\!] - \lim_{x\to 2^-} 5$$

$$= 3\cdot \lim_{x\to 2^-}[\![x]\!] - 5$$

$$= 3\cdot 1 - 5 = -2$$

$f(x) = \cos x$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\cos(x+h) - \cos x}{h}$$

$$= \lim_{h\to 0} \frac{\cos x \cosh - \sin x \sinh - \cos x}{h}$$

$$= \lim_{h\to 0} \frac{\cos x \cosh - \cos x}{h} - \lim_{h\to 0} \frac{\sin x \sinh}{h}$$

$$= \lim_{h\to 0} -\cos x \frac{(1-\cosh)}{h} - \lim_{h\to 0} \sin x \frac{\sinh}{h}$$

$\frac{(1-\cosh)}{h} \to 0$, $\frac{\sinh}{h} \to 1$

$$(-\cos x) \cdot 0 - (\sin x) \cdot 1$$

$$= \boxed{-\sin x}$$

$$\frac{a}{1} \cdot \frac{b}{c} = \frac{a \cdot b}{c}$$

HW

Test 1 Practice Problems!