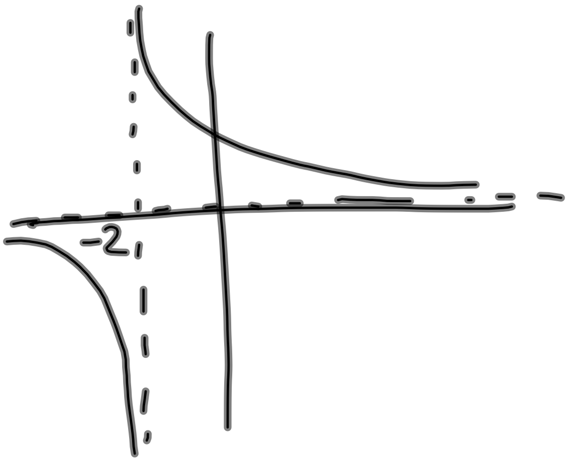
Test 1 Practice Problems

1\. c. $y = \frac{1}{(x+2)}$ $\frac{1}{x}+2$



domain: $(-\infty,-2)\cup(-2,\infty)$

range: $(-\infty,0)\cup(0,\infty)$

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

12. $$\lim_{x\to 0}\frac{\tan^2 x}{2x}$$

$$=\lim_{x\to 0}\frac{\sin^2 x}{2x\cos^2 x}=\lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{\sin x}{2\cos^2 x}$$

$$1 \qquad 0$$

$$=\boxed{0}$$

15. TB Continued...

3. $f(x) = x^2 + 1$ $\quad$ $f \circ g$ & $g \circ f$

$g(x) = \sin x$

$(f \circ g)(x) = \sin^2 x + 1$

$(g \circ f)(x) = \sin(x^2 + 1)$

both have domain $\mathbb{R}$

5. $\lim_{x \to 2} (2x-3) = 1$

$f(x) = 2x-3$

$L = 1 \,;\, c = 2$

Given $\varepsilon > 0$

find $\delta > 0$

Given $\varepsilon > 0$,

$|f(x) - L| =$

$|2x-3-1| = |2x-4| = 2|x-2| < \varepsilon$

$\iff |x-2| < \varepsilon/2$ Take $\boxed{\delta = \varepsilon/2.}$

16. $$\lim_{x\to 0}\left(x^2 \sin\frac{3}{x}\right)+2$$

$$-1 \le \sin\frac{3}{x} \le 1$$

$$-x^2+2 \le x^2\sin\frac{3}{x}+2 \le x^2+2$$

$$0 \qquad\qquad\qquad\qquad 0$$

$$+2=2\cancel{0}$$

$$\Rightarrow \lim_{x\to 0} x^2\sin\frac{3}{x} = 0.$$

17. $$2-3x^2 \le f(x) \le 2+5x^2$$

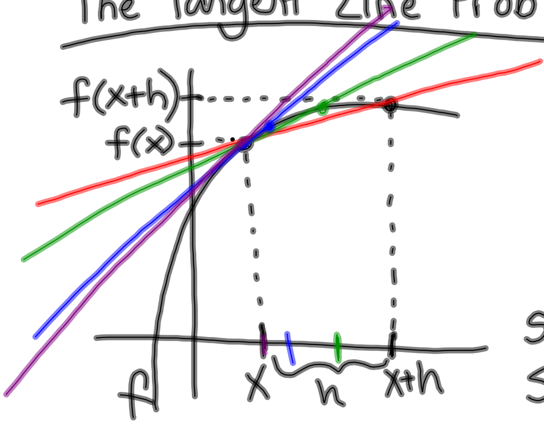
$$2 \qquad\qquad\qquad\qquad 2$$

$$\therefore \lim_{x\to 0} f(x) = 2.$$

9. $\lim\limits_{x \to -1} \dfrac{x^2-9}{x^2-5x+6} =$

$= \lim\limits_{x \to -1} \dfrac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}(x-2)} = \boxed{-\dfrac{2}{3}}$

2.1 The Derivative & The Tangent Line Problem



secant line
crosses through a function at two points

slope of the secant line:

$$\frac{f(x+h)-f(x)}{x+h-x} = \frac{f(x+h)-f(x)}{h}$$

"the difference quotient"

what happens as $h \to 0$?

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

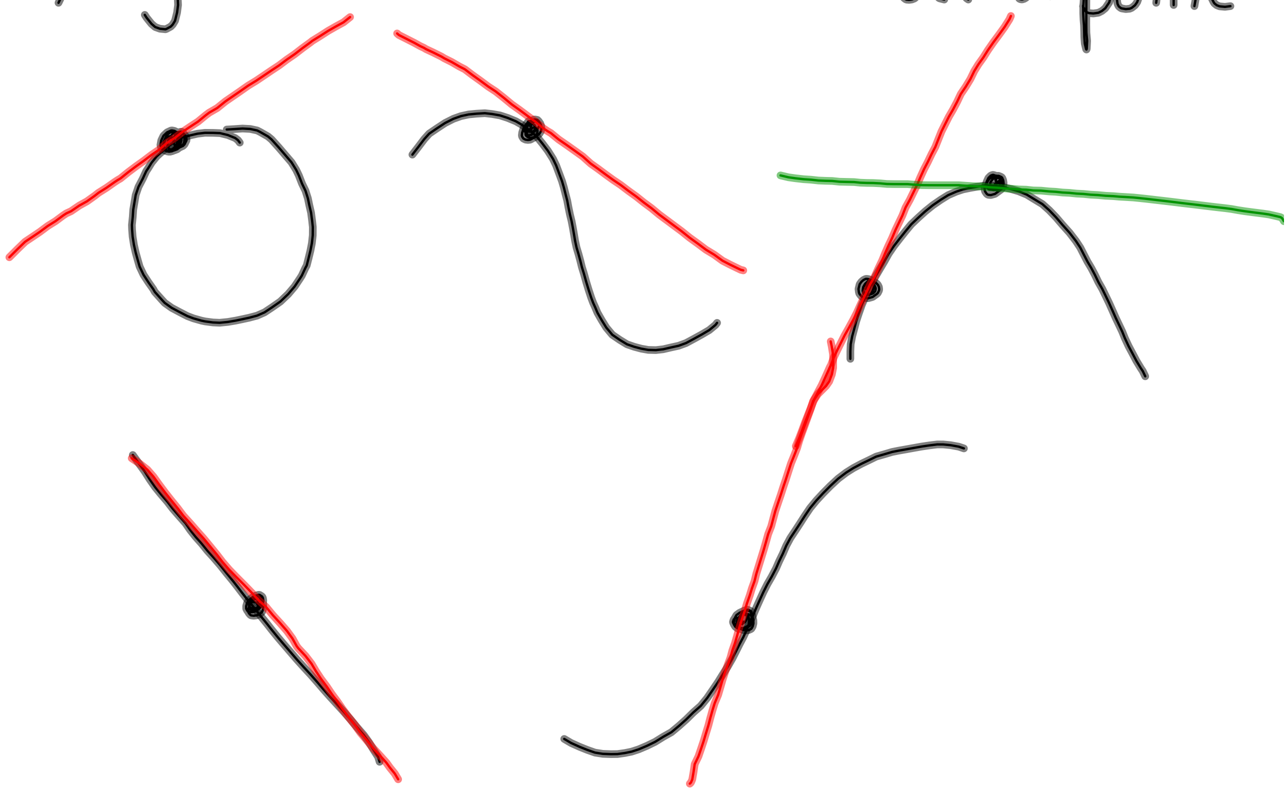
As $h \to 0$, the secant line approximates the tangent line, and the limit is the slope of the tangent line and we call it the derivative of f at x.

$f'(x)$ "f prime of x"

$\frac{dy}{dx}$, y', $\frac{d}{dx}[f(x)]$, $D_x[y]$

"derivative of y with respect to x"
"y prime"
"the derivative w.r.t. x of f(x)"
"partial derivative w.r.t. x of y"

Tangent line to a curve at a point



2.1

$$\boxed{\Delta x = h}$$

8. $g(x) = 5 - x^2$

find slope of tangent line at the points $(2,1)$ & $(0,5)$

$(2,1)$ $x = 2$, $g(x) = 1$

$$\lim_{\Delta x \to 0} \frac{g(x+\Delta x) - g(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{5 - (x+\Delta x)^2 - 1}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{5 - \left(x^2 + 2x\Delta x + (\Delta x)^2\right) - 1}{\Delta x} =$$

$x=2$

$$= \lim_{\Delta x \to 0} \frac{4 - x^2 - 2x\Delta x - (\Delta x)^2}{\Delta x} = \lim_{\Delta x \to 0} \frac{\cancel{4} - \cancel{4} - 4\Delta x - (\Delta x)^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(-4 - \Delta x)}{\cancel{\Delta x}} = \boxed{-4}$$

$(0,5)$ $x = 0$, $f(x) = 5$ $f(x) = 5 - x^2$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{5 - (0+h)^2 - 5}{h}$$

$$= \lim_{h \to 0} \frac{-h^2}{h} = \lim_{h \to 0} -h = \boxed{0}$$

find the derivative

20. $f(x) = x^3 + x^2$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^3 + (x+h)^2 - (x^3 + x^2)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 + \cancel{x^2} + 2xh + h^2 - \cancel{x^3} - \cancel{x^2}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2 + 2x + h)}{\cancel{h}}$$

$$= \boxed{3x^2 + 2x}$$