Test #2 Practice Problems

9. Find y' in terms of $x \,\&\, y$.

$$(x^2y + 3xy^3)' = (5x^3y^2)'$$

$$(5x^3)(2yy')$$

$$2xy + x^2y' + 3y^3 + 9xy^2y' = 15x^2y^2 + 10x^3yy'$$

$$x^2y' + 9xy^2y' - 10x^3yy' = 15x^2y^2 - 2xy - 3y^3$$

$$y' = \frac{15x^2y^2 - 2xy - 3y^3}{x^2 + 9xy^2 - 10x^3y}$$

$$(10y)' = 10y'$$

Old Test 2

2. $V = \frac{4}{3}\pi r^3$

a. instantaneous when $r = 3\text{cm}$

$$\frac{dV}{dr} = 4\pi r^2 \Big|_{r=3} = 4\pi(3)^2 = \boxed{36\pi \text{ cm}^2}$$

b. average rate of change $r = 1\text{cm}$ to $r = 2\text{ cm}$

$$\frac{\Delta V}{\Delta r} = \frac{\frac{4}{3}\pi(2)^3 - \frac{4}{3}\pi(1)^3}{2-1} = \frac{32\pi}{3} - \frac{4\pi}{3} = \boxed{\frac{28\pi}{3} \text{ cm}^2}$$

P.P.

10. find y'. $(\cos x + \sin y)' = (\tan(xy))'$

$-\sin x + y'\cos y = \sec^2(xy) \cdot (y + xy')$

$-\sin x + y'\cos y = y\sec^2(xy) + xy'\sec^2(xy)$

$y'\cos y - xy'\sec^2(xy) = y\sec^2(xy) + \sin x$

$$y' = \frac{y\sec^2(xy) + \sin x}{\cos y - x\sec^2(xy)}$$

$s(t) = \frac{1}{2}at^2 + v_0t + s_0$

$s(t) = -16t^2 + v_0t + s_0$

$v(t) = s'(t) = -32t + v_0$

$a(t) = v'(t) = s''(t) = -32$

P.P.

4. $f(x) = (-3^x)(\cot(5x^2+4x))$

$$f'(x) = -3^x \cdot \ln 3 \cot(5x^2+4x) + -3^x\left(-\csc^2(5x^2+4x)\cdot(10x+4)\right)$$

2.5

13. $(\sin x)' = (x(1+\tan y))'$

$$\cos x = 1+\tan y + x(\sec^2 y \cdot y')$$

$a(b+c) = ab+ac$

$$\frac{\cos x - 1 - \tan y}{x\sec^2 y} = y'$$

31. $((x^2+y^2)^2)' = (4x^2y)'$ find m @ (1,1)

$$2(x^2+y^2)\cdot(2x+2yy') = 8xy + 4x^2y'$$

$$4x(x^2+y^2) + 4yy'(x^2+y^2) = 8xy + 4x^2y'$$

$$4yy'(x^2+y^2) - 4x^2y' = 8xy - 4x(x^2+y^2)$$

$$y' = \left.\frac{8xy-4x(x^2+y^2)}{4y(x^2+y^2)-4x^2}\right|_{x=1,\,y=1} = \frac{8-4(1)(2)}{4(1)(2)-4(1)} = 0$$

35. Find y''. $x^2+y^2=36$

$2x+2yy'=0$

$2yy'=-2x$

$y'=\frac{-2x}{2y}=\frac{-x}{y}=-xy^{-1}$

$y''=(-1)(y^{-1})+(-x)(-y^{-2}\cdot y')$

$=-y^{-1}+xy^{-2}(-xy^{-1})=\frac{-1}{y}-\frac{x^2}{y^3}$

15.

$y=\sin(xy)$

$y'=\cos(xy)\cdot(1\cdot y+x\cdot y')$

$y'=y\cos(xy)+xy'\cos(xy)$

$y'-xy'\cos(xy)=y\cos(xy)$

$\boxed{y'=\frac{y\cos(xy)}{1-x\cos(xy)}}$

p.p.

3. $f(x)=5\sin^2\left(\sqrt{3\csc(7x^2-2x)}\right)$

$f(x)=5\left[\sin\left(3\csc(7x^2-2x)\right)^{1/2}\right]^2$

$f'(x)=10\sin\left(3\csc(7x^2-2x)\right)^{1/2}\cdot\cos\left(3\csc(7x^2-2x)\right)^{1/2}$
$\cdot\frac{1}{2}\left(3\csc(7x^2-2x)\right)^{-1/2}\cdot\left(-3\csc(7x^2-2x)\cot(7x^2-2x)\right)$
$\cdot(14x-2)$

$y=x^3$

$y'=3x^2$

$y''=6x$

$y^{(3)}=6$

$y^{(4)}=0$

for n^{th} deg. polyn.
$n+1^{st}$ derivative is 0.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

For $f(x) = 2x^2 - 3x$,

find the derivative using the limit definition.

$$\lim_{h \to 0} \frac{2(x+h)^2 - 3(x+h) - (2x^2 - 3x)}{h}$$

$$= \lim_{h \to 0} \frac{2(x^2 + 2xh + h^2) - 3x - 3h - 2x^2 + 3x}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 - \cancel{3x} - 3h - \cancel{2x^2} + \cancel{3x}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(4x + 2h - 3)}{\cancel{h}}$$

$$= \boxed{4x - 3}$$