Quiz #5

1. $-5\sin 5x$

2. $\dfrac{\sec^2 x}{\tan x}$

3. $\dfrac{2e^{\arctan 2x}}{1+(2x)^2}$

4. $\frac{1}{3}(\sec x)^{-2/3}\cdot \sec x\tan x$

5. $-12e^{x}\cos(3e^{x})$

1. Find $f'(x)$.

a. $f(x) = \frac{2}{\sqrt{x}} + 3\arccos 2x$

$= 2x^{-1/2} + 3\arccos(2x)$

$f'(x) = 2\left(-\frac{1}{2}x^{-3/2}\right) - \dfrac{3}{\sqrt{1-(2x)^2}}\cdot 2$

$= -x^{-3/2} - \dfrac{6}{\sqrt{1-(2x)^2}}$

b. $f(x) = -2\cos x - 6x^{-1/4} + 3$

$f'(x) = -2(-\sin x) - 6\left(\frac{-1}{4}x^{-5/4}\right)$

$= 2\sin x + \frac{3}{2}x^{-5/4}$

2. Find $f'(x)$.

$f(x) = [\sec(\ln x)][\ln(\sec x)]$

$[\sec(\ln x)]\cdot[\ln(\sec x)]' + [\ln(\sec x)][\sec(\ln x)]'$

$[\sec(\ln x)]\cdot \dfrac{1}{\cancel{\sec x}}\cdot \cancel{\sec x}\tan x + [\ln(\sec x)]\cdot \sec(\ln x)\tan(\ln x)\cdot\dfrac{1}{x}$

$\sec(\ln x)\tan x + \dfrac{\ln(\sec x)\sec(\ln x)\tan(\ln x)}{x}$

3. Find $f'(x)$.

$f(x) = \frac{3e^x}{\arctan 2x} = 3e^x(\arctan 2x)^{-1}$

$$f'(x) = \frac{(\arctan 2x)(3e^x) - (3e^x)\cdot\frac{1}{1+(2x)^2}2}{(\arctan 2x)^2}$$

$$f'(x) = 3e^x\cdot\left[-(\arctan 2x)^{-2}\cdot\frac{1}{1+(2x)^2}\cdot 2\right] + 3e^x(\arctan 2x)^{-1}$$

$$\cancel{-\left(\frac{1}{1+(2x)^2}\right)^{-2}}$$

4. Find $f'(x)$.

a. $f(x) = -3\sqrt{\cot(4-7x)}$

$= -3[\cot(4-7x)]^{1/2}$

$$f'(x) = -\frac{3}{2}[\cot(4-7x)]^{-1/2}\cdot(-\csc^2(4-7x))\cdot(-7)$$

b. $f(x) = \csc^2(\log_3 x) = [\csc(\log_3 x)]^2$

$$2\csc(\log_3 x)\cdot(-\csc(\log_3 x)\cot(\log_3 x))\cdot\frac{1}{x\ln 3}$$

$$= \frac{-2\csc^2(\log_3 x)\cot(\log_3 x)}{x\ln 3}$$

5. Find $f''(x)$.

$f(x) = 7^{\sin(3x-5)}$

$(5x^3)(3\sqrt{x-1})$ $[cf(x)]' = cf'$

$$f'(x) = 7^{\sin(3x-5)}\cdot\ln 7\cdot\cos(3x-5)\cdot 3$$

$$= (3\ln 7)\cdot 7^{\sin(3x-5)}\cos(3x-5)$$

$$f''(x) = (3\ln 7)7^{\sin(3x-5)}(-\sin(3x-5)\cdot 3) + \cos(3x-5)\cdot$$

$$(3\ln 7)\cdot 7^{\sin(3x-5)}\cdot\ln 7\cos(3x-5)\cdot 3$$

6. Find $f'(x)$ using the limiting definition.

$f(x) = x^2 - x - 3$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{(x+h)^2-(x+h)-3-(x^2-x-}{h}$$

$$= \lim_{h\to 0}\frac{\cancel{x^2}+2xh+h^2\cancel{-x}-h\cancel{-3}\cancel{-x^2}\cancel{+x}\cancel{+3}}{h}$$

$$= \lim_{h\to 0}\frac{\cancel{h}(2x+h-1)}{\cancel{h}} = \boxed{2x-1}$$

$(fg)' = fg' + f'g$ $\qquad ([f(x)]^2)' = 2f(x)\cdot f'(x)$

7. Find y' implicitly in terms of x and y.
$xy + y^2 = 3x$

$xy' + 1\cdot y + 2y\cdot y' = 3$

$y'(x+2y) = 3 - y$

$y' = \dfrac{3-y}{x+2y}$

8. Find y' implicitly in terms of x and y.
$\cos y + \tan x = \sin y$

$-\sin y \cdot y' + \sec^2 x = \cos y \cdot y'$

$\sec^2 x = y'(\cos y + \sin y)$

$y' = \dfrac{\sec^2 x}{\sin y + \cos y}$

9. Find the equation of the tangent line to the graph of $f(x) = -\frac{6}{\sqrt[3]{x^2}} + 1$ at the point $(1,-5)$. x_1, y_1
Give your answer in the form $y = mx + b$.

$f(x) = -6x^{-2/3} + 1$

$f'(x) = -6\left(\frac{-2}{3}\right)x^{-5/3}$

$m = f'(1) = -6\left(\frac{-2}{3}\right) = 4$

$y - y_1 = m(x - x_1)$

$y - (-5) = 4(x-1)$

$y + 5 = 4x - 4$

$\boxed{y = 4x - 9}$

10. Given that the surface area of a sphere is $A = 4\pi r^2$,
a. Find the instantaneous rate of change of surface area with respect to radius length when the radius is 2 cm. Give an exact, simplified answer in terms of π.

$A(r) = 4\pi r^2$

$A'(r) = 8\pi r \Big|_{r=2\text{cm}} = 8\pi(2) = \boxed{16\pi \text{ cm}}$

b. Find the average rate of change of surface area as the radius changes from 1 cm to 3 cm. Give an exact, simplified answer in terms of π.

$\dfrac{\Delta A}{\Delta r} = \dfrac{4\pi(3)^2 - 4\pi(1)^2}{3-1} = \dfrac{36\pi - 4\pi}{2}$

$= \dfrac{32\pi}{2} = \boxed{16\pi \text{ cm}}$

Bonus A: Determine the point(s) (if any) at which the graph of the function has a horizontal tangent line.
$y = x^3 - 3x^2 + 5$

$y' = 3x^2 - 6x$

$3x(x-2) = 0$

$x = 0, x = 2$

$\boxed{(0,5) \& (2,1)}$

Bonus B: Find the polynomial $P_2(x) = a_0 + a_1 x + a_2 x^2$ whose value and first two derivatives agree with the value and first two derivatives of $f(x) = \sin x$ at the point $x = 0$. This polynomial is called the second-degree Taylor polynomial of $f(x) = \sin x$ at $x = 0$.

$P_2(x) = a_0 + a_1 x + a_2 x^2$ $\quad f(x) = \sin x$

$P_2(0) = a_0$ $\quad a_0 = 0$ $\quad f(0) = \sin 0 = 0$

$P_2'(x) = a_1 + 2a_2 x$ $\quad f'(x) = \cos x$

$P_2'(0) = a_1$ $\quad a_1 = 1$ $\quad f'(0) = \cos 0 = 1$

$P_2''(x) = 2a_2$ $\quad f''(x) = -\sin x$

$P_2''(0) = 2a_2$ $\quad a_2 = 0$ $\quad f''(0) = -\sin 0 = 0$

$P_2(x) = a_0 + a_1 x + a_2 x^2$

$\boxed{P_2(x) = x}$