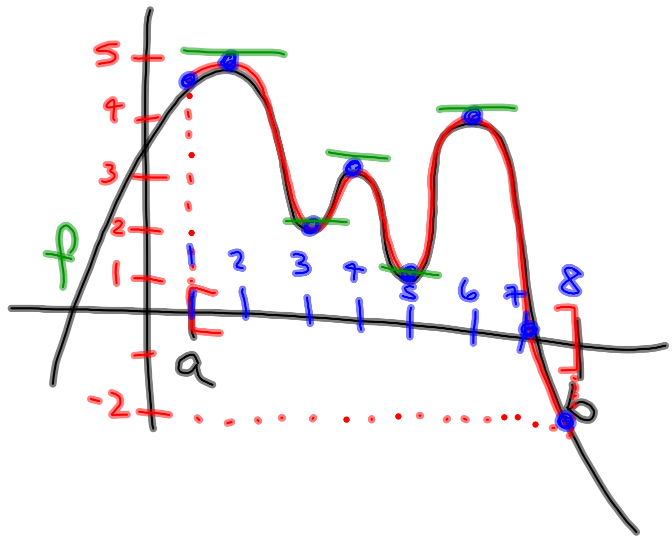
3.1 Extrema on an Interval

- maxima & minima
- relative & absolute



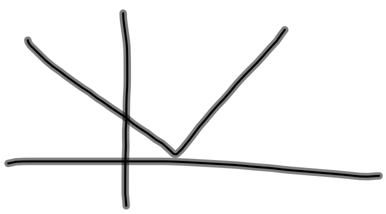
relative minima:
(3,2), (5,1)

relative maxima:
(2,5), (4,3), (6,4)

absolute maximum:
5 @ (2,5)

absolute minimum:
−2 @ (8,−2)

f(x) has a relative maximum or minimum when $f'(x) = 0$. or



$f'(x)$ is undefined.

We call such x-values

Critical #'s of f.

3.1

28. $h(t) = \frac{t}{t-2}$, $[3,5]$

$$h'(t) = \frac{(t-2)\cdot 1 - t(1)}{(t-2)^2} = \frac{-2}{(t-2)^2}$$

$h'(t)$ is never $=0$

$h'(t)$ is undefined @ $t=2$

$\Rightarrow$ 2 is our only critical #

$2 \notin [3,5]$ so who cares about it

$h(3) = \frac{3}{3-2} = \frac{3}{1} = 3$ abs max

$h(5) = \frac{5}{5-2} = \frac{5}{3}$ abs. min

30. $g(x) = \sec x$, $\left[\frac{-\pi}{6}, \frac{\pi}{3}\right]$

$g'(x) = \sec x \tan x$

$$\frac{2}{\frac{2}{\sqrt{4}}} \le \frac{2}{\sqrt{3}} \le \frac{2}{\frac{1}{\sqrt{1}}}$$

$\sec x \tan x = 0$

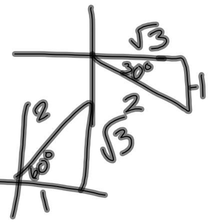
$\sec x = 0$ never! $\tan x = 0$

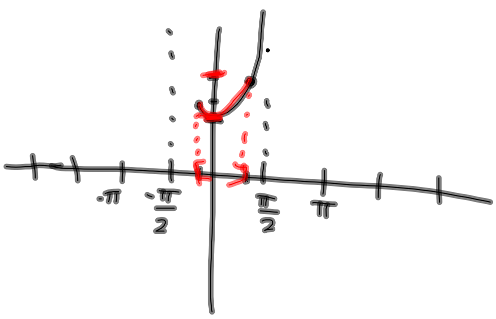
$x = 0 \leftarrow$ only critical # in interval!

$g\left(\frac{-\pi}{6}\right) = \sec\left(\frac{-\pi}{6}\right) = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$g(0) = \sec(0) = 1$ abs min

$g\left(\frac{\pi}{3}\right) = \sec\left(\frac{\pi}{3}\right) = 2$ abs max





22. $f(x) = x^3 - 12x$, $[0, 4]$

$\boxed{\frac{3.1\text{ HW}}{17\text{-}31\text{ odd}}}$

$f'(x) = 3x^2 - 12$

$3(x^2 - 4) = 0$

$3(x-2)(x+2) = 0$

$x = 2, -2$

not in interval

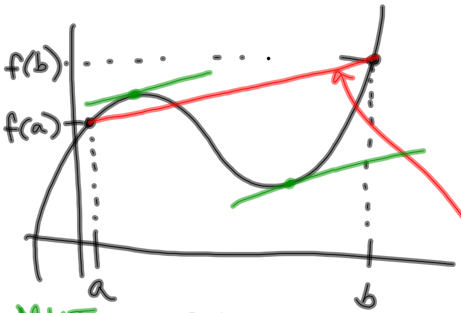
f has critical #'s 2 & -2

$f(0) = 0^3 - 12(0) = 0$

$f(2) = 2^3 - 12(2) = 8 - 24 = -16$ abs. min

$f(4) = 4^3 - 12(4) = 64 - 48 = 16$ abs max

3.2 Rolle's Theorem & The Mean Value Theorem



Carter-Gelsinger eversion

"Hairy Ball Theorem"

secant line

slope: $\frac{f(b)-f(a)}{b-a}$

MVT says:

If f is continuous on $[a,b]$ and differentiable on (a,b), then there exists at least one $c \in (a,b)$ such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

Rolle's Theorem is special case where $f(a) = f(b)$ and hence a horizontal ~~secant~~ tangent line.

$\exists\, c \in (a,b)$ s.t. $f'(c) = 0$.

Can Rolle's Thm be applied?
If so, find all c's in $[a,b]$
s.t. $f'(c)=0$.

8. $f(x)=x^2-5x+4$, $[1,4]$

f cts on $[1,4]$? yes
f diff. on $(1,4)$? yes
Is $f(a)=f(b)$? yes
$f(1)=1-5+4=0$
$f(4)=16-20+4=0$

Rolle's Theorem <u>does</u> apply.

$f'(x)=2x-5$
$2x-5=0$
$2x=5$
$x=5/2$

$\boxed{5/2} \in [1,4]$

3.2
7-19 odd