Review

Determine if Rolle's Theorem applies, and if so, find all values it guarantees.

$f(x) = \frac{x+3}{x^2}$; $[1,3]$

Is f cts on $[1,3]$? Yes!

diff on $(1,3)$? Yes!

Is $f(a) = f(b)$? no

Rolle's Thm does not apply!

3.2

$f(x) = x^2$, $[-2,1]$

$$\frac{f(b)-f(a)}{b-a} = \frac{1^2-(-2)^2}{1-(-2)} = \frac{1-4}{3} = -1$$

$f'(x) = 2x$

$2x = -1$

$x = -\frac{1}{2}$

- Find critical #'s & open intervals on which f is increasing/decreasing
- Find inflection points & intervals on which f is concave up/down

(absolute/relative extrema)

3.3

16. $f(x) = x^3 - 6x^2 + 15$

$f'(x) = 3x^2 - 12x$

$3x(x-4) = 0$

$x = 0, 4$ ← critical #'s

relative extrema: $(0, 15)$ & $(4, -17)$
max min

$f'(-1)$, $f'(1)$, $f'(5)$

$+ \quad 0 \quad - \quad 4 \quad +$

f is increasing on $(-\infty, 0) \cup (4, \infty)$
decreasing on $(0, 4)$

3
16
6
96

64 - 96 + 15
79
17

3.4

16. $f(x) = x^3(x-4) = x^4 - 4x^3$

$f'(x) = 4x^3 - 12x^2$

$4x^2(x-3) = 0$

$x = 0, 3$ critical #'s

$f'(-1)$, $f'(1)$, $f'(4)$

$- \quad 0 \quad - \quad 3 \quad +$

$\searrow \quad \searrow \quad \nearrow$

f is decreasing on $(-\infty, 0) \cup (0, 3)$

increasing on $(3, \infty)$

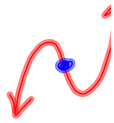
f has a minimum @ $(3, -27)$

$f''(x) = 12x^2 - 24x$

$12x(x-2) = 0$

$x = 0, 2$

$\Rightarrow$ inflection points @ $(0,0)$ & $(2, -16)$



$f''(-1)$, $f''(1)$, $f''(3)$

$+ \quad 0 \quad - \quad 2 \quad +$

f is concave up on $(-\infty, 0) \cup (2, \infty)$

concave down on $(0, 2)$

3.3

30. $f(x) = \dfrac{x+3}{x^2}$

$$f'(x) = \frac{x^2(1) - (x+3)(2x)}{(x^2)^2} = \frac{x^2 - 2x^2 - 6x}{x^4}$$

$$= \frac{-x^2 - 6x}{x^4} = \frac{-x(x+6)}{x^4}$$

undefined @ 0

$-x^2 - 6x = 0$

$-x(x+6) = 0$

$x = -6$

critical #'s: 0 & -6

$f'(-7)$, $f'(-1)$, $f'(1)$

$- \quad -6 \quad + \quad 0 \quad -$

$\searrow \quad \nearrow \quad \searrow$

relative min: $\left(-6, -\frac{1}{12}\right)$

~~relative max (0,~~

f is increasing on $(-6, 0)$

f is decreasing on $(-\infty, -6) \cup (0, \infty)$

3.4 #20

$(x^{1/2})' = \frac{1}{2}x^{-1/2}$

$f(x) = \dfrac{x+1}{\sqrt{x}}$

$f'(x) = \dfrac{\sqrt{x}(1)\frac{2\sqrt{x}}{2\sqrt{x}} - (x+1)\cdot\frac{1}{2\sqrt{x}}}{(\sqrt{x})^2}$

$= \dfrac{\frac{2x - (x+1)}{2\sqrt{x}}}{\frac{x}{1}} = \dfrac{x-1}{2x^{3/2}}$

$x^m x^n = x^{m+n}$

$f'(x) = \dfrac{x-1}{2x^{3/2}}$

$-3x^{3/2} + 3x^{1/2}$

$f''(x) = \dfrac{2x^{3/2}(1) - (x-1)\cdot 3x^{1/2}}{4x^3}$

$= \dfrac{x^{1/2}(2x^{2/2} - 3x^{2/2} + 3)}{4x^3}$

$f''(x) = \dfrac{x^{1/2}(3-x)}{4x^3}$

inflection pts: $\left(3, \frac{4}{\sqrt{3}}\right)$

$\cancel{f''(-1)}$ $f''(1)$ $f''(4)$

0 + 3 −

f is concave up on $(0,3)$

down on $(3,\infty)$

3.3

#11-31

3.4

#11-25

Old Test #3

(+ ~~2nd~~ concavity)

Practice Probs #3

(− l'Hopitals Rule